Dynamic Mechanism Design

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Plan for the talk

- Review a few strands of literature
  - Buyers with *independent* values over time (additive)
  - Buyers with values *evolving* over time (additive)
  - Buyers with *fixed value* over time (additive)
  - Buyers with *fixed value, but unit-demand / fixed budget*, and unknown supply
- Discuss main results & commonly used techniques
- Present future directions / open problems

Is independence justified?

- Mechanisms for repeated interactions between seller and buyer (eg. Internet ad auctions)

- Buyers with independent values over time
    - Eg. Ad impressions arrive over time
    - Value distribution is a function of (age, location, gender,…)
    - Usually independent across time

General questions

- General questions:
  - Best achievable revenue and welfare?
  - Compare with single-shot optimal
  - Is the mechanism easy to implement?
  - What flavor of IC/IR does it satisfy?
- State of the art in real-world:
  - Classic single-shot auctions have found their way to the web

Independent values model

- Single buyer (many results extend to multiple buyers)

- For steps $t = 1 \dots T$
  - item arrives (ad impression)
  - Buyer observes his value $v_t \sim F_t$
  - Buyer reports bid b_t
  - Auction decides allocation $x_t(b_{1\dots t}, F_{1\dots T})$ and payment $p_t(b_{1\dots t}, F_{1\dots T})$
  - Buyer gets utility: $u_t = v_t x_t(b_{1\dots t}, F_{1\dots T}) - p_t(b_{1\dots t}, F_{1\dots T})$

- Buyer wants to maximize overall utility:

$$U_t = u_t(v_t, b_{1\dots t}, F_{1\dots T}) + E_{F_{t+1\dots T}}[\sum_{\tau=t+1}^{T} u_\tau(v_\tau, b_{1\dots\tau}, F_{1\dots T})]$$

- Let $h_t = b_{1\dots t}$ and let $\boldsymbol{F} = F_{1\dots T}$

Constraints

- Dynamic Incentive Compatibility:

$$\forall\, t, h_{t-1}:\ \ v_t \in argmax_b\, U_t(v_t, (h_{t-1}, b), \boldsymbol{F})$$

- Dynamic Individual Rationality:

$$\forall\, t, h_{t-1}:\ \ U_t(v_t, (h_{t-1}, v_t), \boldsymbol{F}) \geq \mathbf{0}$$

- Per round / periodic Individual Rationality:

$$\forall\, t, h_{t-1}:\ \ v_t x_t((h_{t-1}, v_t), \boldsymbol{F}) - p_t((h_{t-1}, v_t), \boldsymbol{F}) \geq 0$$

Why link auctions across time?

- After all, single-shot auctions are:
  - easy to reason about for buyers
  - easy to implement for sellers
- Motivation:
  - Better targeting technologies → more surplus to buyers
  - Auctions are quite thin → not much competition
  - Need ways to improve publisher revenue

Do we benefit by linking?

- Sell today, a single item whose value in U[0,1] will be realized tomorrow
  - Post price = ½$-\epsilon$ today: buyer accepts; revenue = ½$-\epsilon$
  - But violates ex-post IR
  - Post price = ½ tomorrow: buyer accepts when $v \geq \frac{1}{2}$, revenue = ¼
  - Can't get more than ¼ with ex-post IR

Do we benefit by linking?

- One item today and one will arrive tomorrow, both U[0,1]:
  - Buyer knows today's value, but not tomorrow's
  - Post price = 1 today; buyer buys if today's $v \geq \frac{1}{2}$
  - This again violates ex-post IR
  - Seems like no benefit from linking?

Unbounded separation

From Papadimitriou+Pierrakos+Psomas+Rubenstein'16:

- Example:
  - Round-1: Equal revenue distribution supported in [1,n]:
    - $F(x) = 1 - \frac{1}{x}$; Mean = $\log(n)$
  - Round-2: Equal revenue distribution supported in $[1, e^n]$
    - Mean = n
- Optimal static auction revenue = 2 (post any price in each round)
- Dynamic mechanism:
  - Allocate always in 1st round, and charge bid b_1
  - Allocate in 2nd round with probability $\frac{b_1}{n}$
  - Utility of bidding b_1: $v_1 - b_1 + \frac{b_1}{n} \cdot n = v_1$ (hence truthful)
  - Revenue = $\log(n)$

Optimal mechanisms

- Papadimitriou+Pierrakos+Psomas+Rubenstein'16
  - Opt. deterministic auction: NP-hard when the days are correlated
  - Opt. randomized auction: computed via LP polynomial in support size

Optimal mechanisms + approximation

- Ashlagi+Daskalakis+Haghpanah'16, Mirrokni+Paes-Leme+Tao+Zuo'16a,'16b:
  - Structural characterization of optimal auction
  - Optimal allocation & payment in round t depend just on a state variable, and round t bid
    - I.e., all other aspects of history irrelevant
  - Optimal auction gives zero utility to buyer in all but last round
  - Give simple constant factor approximations
- Drawback: use positive transfer to get round per-round ex-post IR
  - Extreme example: buyer pays bid (=value) in all but last round where the mechanism compensates him

Martingale utilities

- Real ad auctions: today ~ tomorrow;
  - zero utility for a sequence of days is unacceptable

- Requirement: buyer utility per auction is a martingale [Balseiro+Mirrokni+Paes-Leme'16]
  - Akin to industry practice of smooth delivery/pacing

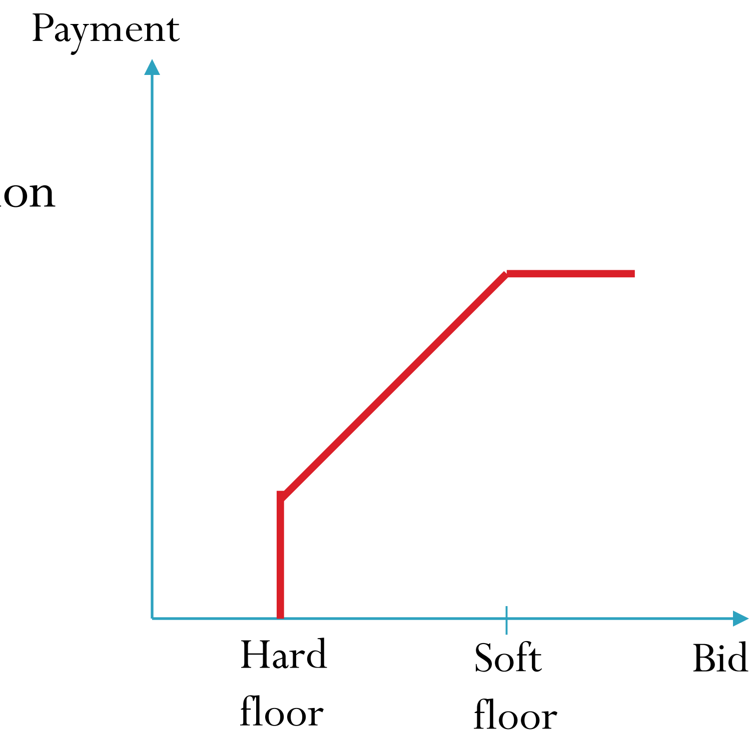
- Model
  - Time discounted infinite horizon model: discount of $\beta \in (0,1)$
  - IID values for buyer across rounds

- Result:
  - Achieve close to entire surplus as the number of rounds $T \to \infty$
  - Simple auction based on hard and soft floors

Hard and soft floors

- Auction:
  - If bid < hard-floor: no allocation
  - Hard-floor < bid < soft-floor: first-price-auction
  - Bid > soft-floor: second-price-auction

- Used in practice by different ad exchanges



Promised utility framework

- Maintain a state variable w_t
  - $x: v_t \times w_t \to [0,1]$ (allocation)
  - $p: v_t \times w_t \to R$ (payment)
  - $u: v_t \times w_t \to w_{t+1}$ (promised utility)

- In round t, apart from allocation $x_{w_t}(v_t)$ and payment $p_{w_t}(v_t)$, mechanism promises a *future discounted utility of $\beta u_{w_t}(v_t)$*

Constraints:

- Dynamic IC:

$$v x_w(v) - p_w(v) + \beta u_w(v) \geq v x_w(v') - p_w(v') + \beta u_w(v')$$

- Promise keeping:

$$w = E_v[v x_w(v) - p_w(v) + \beta u_w(v)]$$

- Dynamic IR:

$$v x_w(v) - p_w(v) + \beta u_w(v) \geq 0$$

Constraints contd…

<u>Constraints:</u>

- Dynamic IC:

$$vx_w(v) - p_w(v) + \beta u_w(v) \geq vx_w(v') - p_w(v') + \beta u_w(v')$$

- Promise keeping:

$$w = E_v[vx_w(v) - p_w(v) + \beta u_w(v)]$$

- Dynamic IR:

$$vx_w(v) - p_w(v) + \beta u_w(v) \geq 0$$

- Periodic IR:

$$vx_w(v) - p_w(v) \geq 0$$

- Martingale:

$E_v[vx_w(v) - p_w(v)]$ is a martingale

Or equivalently

$E_v[u_w(v)]$ is a martingale

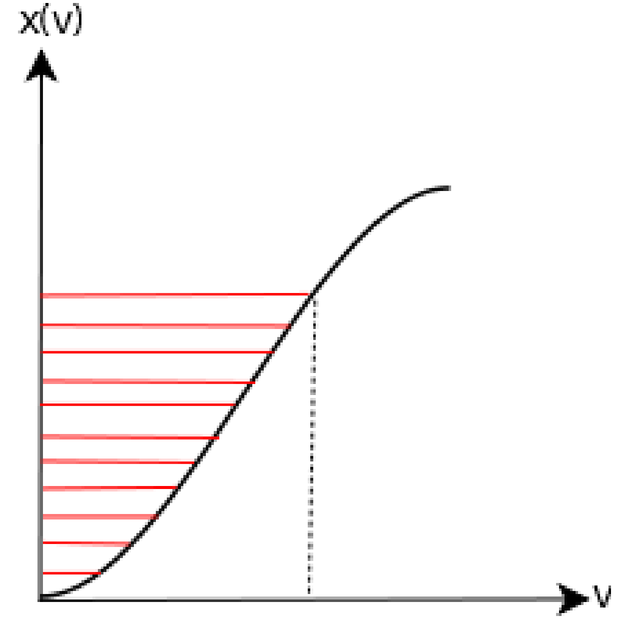
Myerson's technique

Myerson's payment identity:

$$p_w(v) = vx_w(v) - \int_0^v x_w(y)dy$$

Payment identity for our problem:

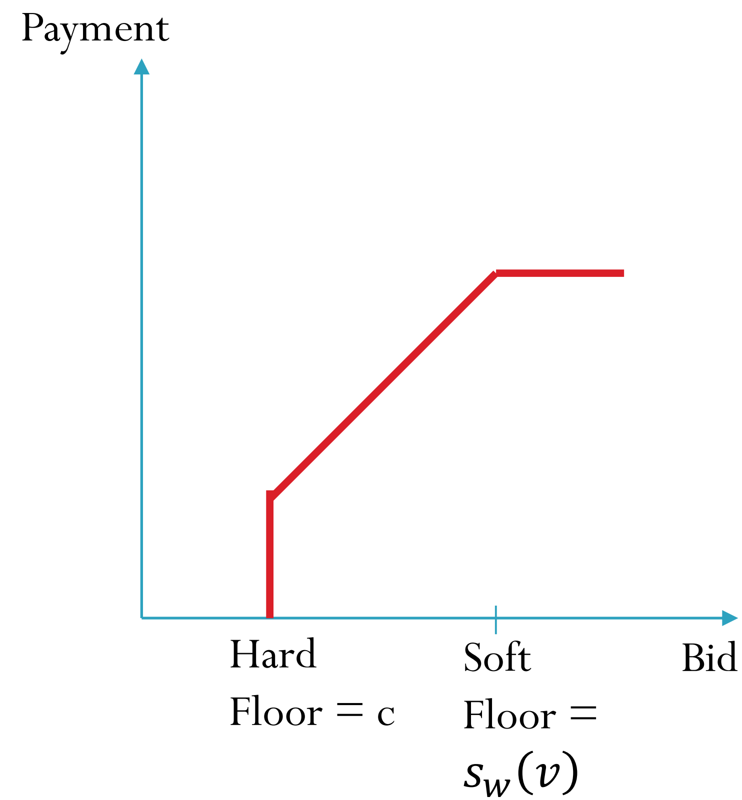
$$p_w(v) - \boldsymbol{\beta u_w(v)} = vx_w(v) - \int_0^v x_w(y)dy$$



Martingale mechanism

The final mechanism:

- Pick state thresholds w_{low} and w_{max}
- When $w \in [w_{low}, w_{max}]$: follow the fixed hard-floor + dynamic soft-floor mechanism
- When $w < w_{low}$: don't allocate



What if buyers don't trust?

- Single-shot IC is easy to verify:
  - Split traffic randomly across k buckets
  - Try different bid shading factors in each bucket
  - Shading factor of 1 should yield highest surplus
- Dynamic IC: impossible to verify
- Buyers:
  - May not trust the seller to stick to his word forever
  - May not be sophisticated
  - May employ learning mechanisms to bid
  - Is your auction robust to all these?

Dynamic IC is fragile

- Dynamic IC:
  - Truth-telling maximizes *current* + *sum-of-all-future-utilities*
  - It assumes all buyers have infinite lookahead
  - Buyer may think seller won't be around for that long!
- What if buyers are limited lookahead: say k-lookahead?
- What if buyers are learners?
  - IC buyers look ahead
  - No-regret learners look back

Robust dynamic auctions?

Agrawal+Daskalakis+Mirrokni+Sivan'17:

- Design a single auction that gets a const. fraction of optimal revenue from
  - a k-lookahead buyer for each k
  - a no-regret learner
  - a policy-regret learner (preferred regret notion against adaptive adversary)
- Setting:
  - Single buyer IID private values drawn repeatedly from a known distribution F

Robust dynamic auctions

- What is the benchmark?
  - Against infinite lookahead buyer, cannot extract more than mean μ
  - Against myopic (0-lookahead) buyer, cannot extract more than R^{Mye}
    - R^{Mye} is revenue of static single-shot revenue optimal mechanism

Robust dynamic auctions

- Result: There exists a single auction that gets, for any $\alpha \in (0,1)$:
  - $(1-\alpha)\mu$ revenue against a k-lookahead buyer *for any* $k \geq 1$
  - $\frac{\alpha}{2} R^{Mye}$ revenue against a myopic buyer
  - $(1-\alpha)\mu$ revenue against a policy-regret learner
  - $\frac{\alpha}{2} R^{Mye}$ revenue against a no-regret learner
- Eg. Choose $\alpha = \frac{2}{3} \rightarrow$ Get a 1/3 approximation against all categories

Matching lower bound

- Result: Any mechanism that gets, for any $\alpha \in (0,1)$, a revenue of
  - $(1-\alpha)\mu$ revenue against an infinite-lookahead buyer
  - Cannot get more than $2\alpha R^{Mye}$ revenue against a myopic buyer

Proof idea for lower bound

- Result: Any mechanism that gets, for any $\alpha \in (0,1)$, a revenue of
  - $(1-\alpha)\mu$ revenue against an infinite-lookahead buyer
  - Cannot get more than $2\alpha R^{Mye}$ revenue against a myopic buyer

1. There are distributions for which:
   - High revenue against myopic buyer → high utility for myopic buyer

2. Infinite-lookahead buyer utility smaller than $\alpha\mu$

3. So myopic buyer utility smaller than $\alpha\mu$

Open questions

1. Extend results to multi-parameter settings
2. Get rid of positive transfer assumption present in many works
3. Make auctions learnable by buyers: IC → learning
4. Can seller learn distributions over time instead of knowing it ahead?
   - What if buyers and seller both play learning algorithms?

Theory still not mature enough to inform practice…

Plan for the talk

Mechanisms for repeated interactions between seller and buyer (eg. Internet ad auctions)

- Review a few strands of literature
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  - Buyers with values *evolving* over time (additive)
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- Discuss main results & commonly used techniques

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Quasilinear utility model

Q: How does a consumer respond to prices?

Standard answer:

1. Consumer has a private value for an item (or bundle of items)
2. His utility for a bundle B is $u(B) = value(B) - price(B)$
3. Consumer picks $B^* \in argmax_B u(B)$

Key assumption: Consumer precisely knows his value for all available bundles

Current selling mechanism: A Buy-It-Now (BIN) price

1. How do consumers make decisions when their private information evolves with usage?

2. How do we use this to improve revenue and buyer utility?

Usage-based value evolution

Chawla+Devanur+Karlin+Sivan'16:

V_0 : Value for first usage; sampled from distribution F

- V_0 is private to buyer
- F is known to seller

V_t : Value for the $t+1$-th usage

- V_t known to buyer only after t usages
- V_t evolves according to a random process known to buyer and seller

- Buy-It-Now (BIN) scheme:

1. Set $\mathbf{E}[\sum_t V_t \mid V_0]$ to be the buyer's "one-shot value"
2. Set the optimal price for this value distribution

- Hurts both the buyer and the seller

Usage-based value evolution

Buy-It-Now Scheme: Price game at $50

Hurts both buyer and seller

1. Buyer: Large payment upfront + Uncertainty about value derived from product
2. Seller:
   - No price discrimination
   - Reducing to "one-shot value" overlooks other natural pricing schemes

Question: What benefits can alternate payment schemes offer, in terms of

a. Revenue
b. Buyer utility
c. Mitigating buyer's risk?

Alternate Payment Schemes

Pay-Per-Play (PPP)

- Seller sets price p_t for t-th usage
- Buyer accepts or rejects p_t
- If buyer rejects, game ends: value stops evolving

E.g. PPP-CAP

- Pay $1/hour
- After paying $50, game is yours

Advantages:

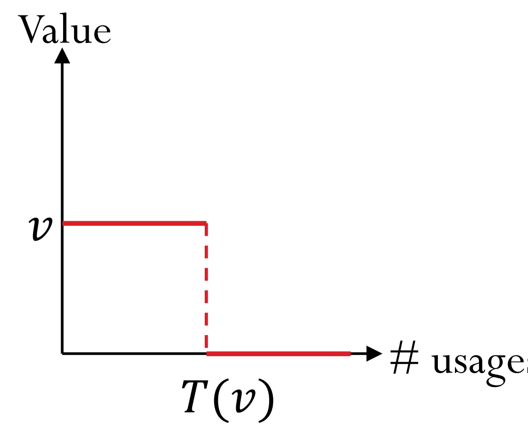
1. *Every* buyer is happier with this scheme than a BIN with $50 price
2. Buyer gets fine-grained control over his utility: never suffer a large "regret"
3. Natural price discriminator
   - Customers who remain interested for a longer time pay more

Revenue?

A warm-up model

<u>Buyer's value:</u>

- $v \in [0,1]$ for the first $T(v)$ usages, and 0 after that
- $T(v)$ is a random variable, with $\mathbf{E}[T(v) \mid v]$ non-decreasing in v



<u>Buyer knows:</u>

- v and $\mathbf{E}[T(v) \mid v]$
- Value for t-th usage only after $t - 1$ usages

<u>Seller knows:</u>

- The distribution F from which v is initially drawn
- The distribution of $T(x)$ for each x

Goal: Compare BIN and PPP

Risk

<u>Risk-neutral buyer:</u>

- Buy whenever expected utility is non-negative

<u>Risk-averse buyer:</u>

- Buy only when probability of negative utility is 0

BIN for risk-neutral buyer

- One-shot value of a risk-neutral buyer with value v is $v \cdot \mathbf{E}[T(v) \mid v]$
- Let $\tilde{F}$ be the distribution of $v \cdot \mathbf{E}[T(v) \mid v]$
- Optimal BIN price $\mathrm{p}^* \in argmax_{\mathrm{p}}\ \max p(1 - \tilde{F}(p))$
- Optimal risk-neutral BIN revenue $= R_0^{BIN} = p^*(1 - \tilde{F}(p^*))$
- [Myerson'81]

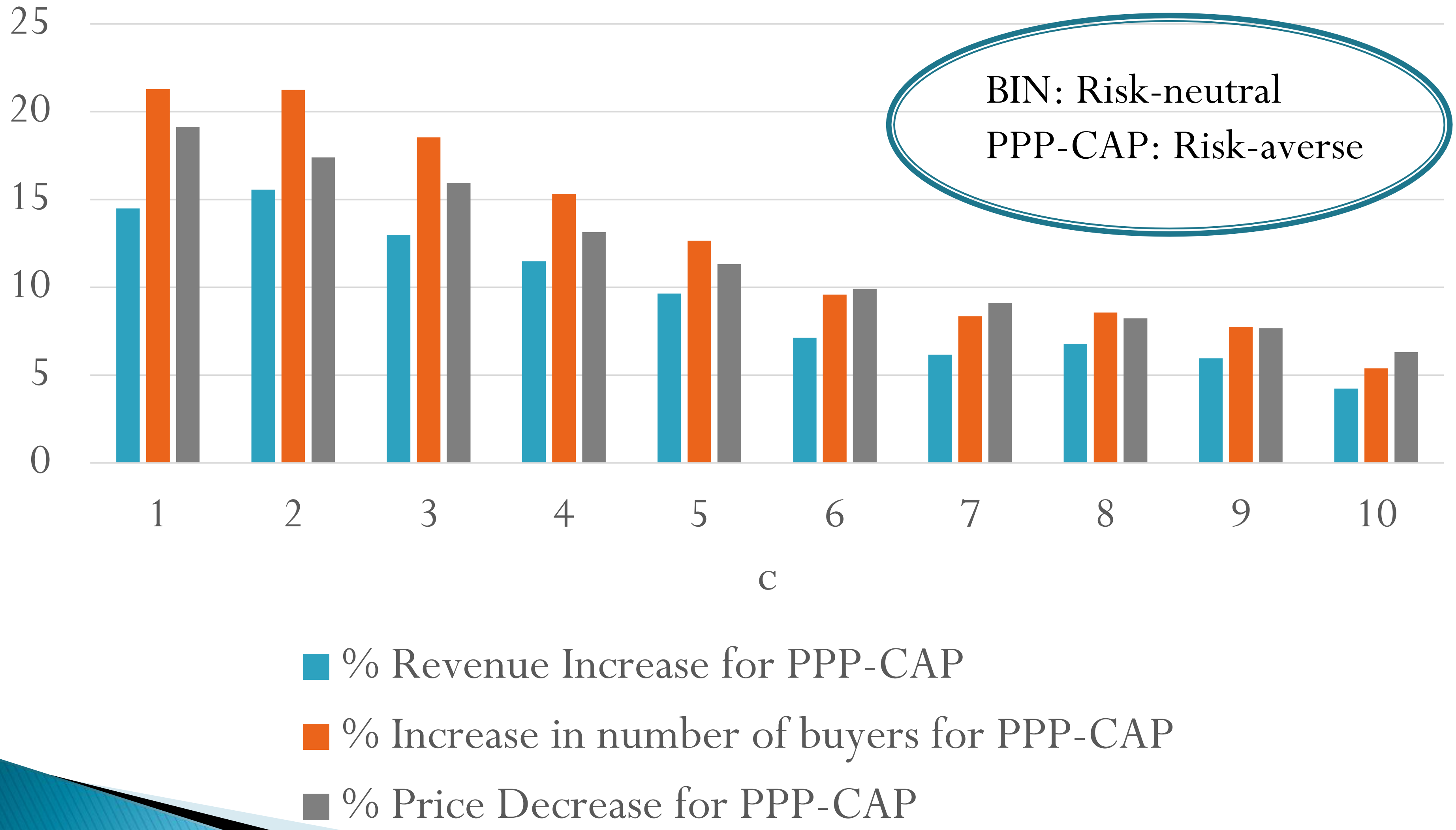
PPP for infinitely risk-averse buyer

- Consider a per-play price of v^*
  - where $v^*\mathbf{E}[T(v^*) \mid v^*] = p^*$is the BIN optimal price
- In b
  - =
- Ho
  - B cheme
  - B cheme

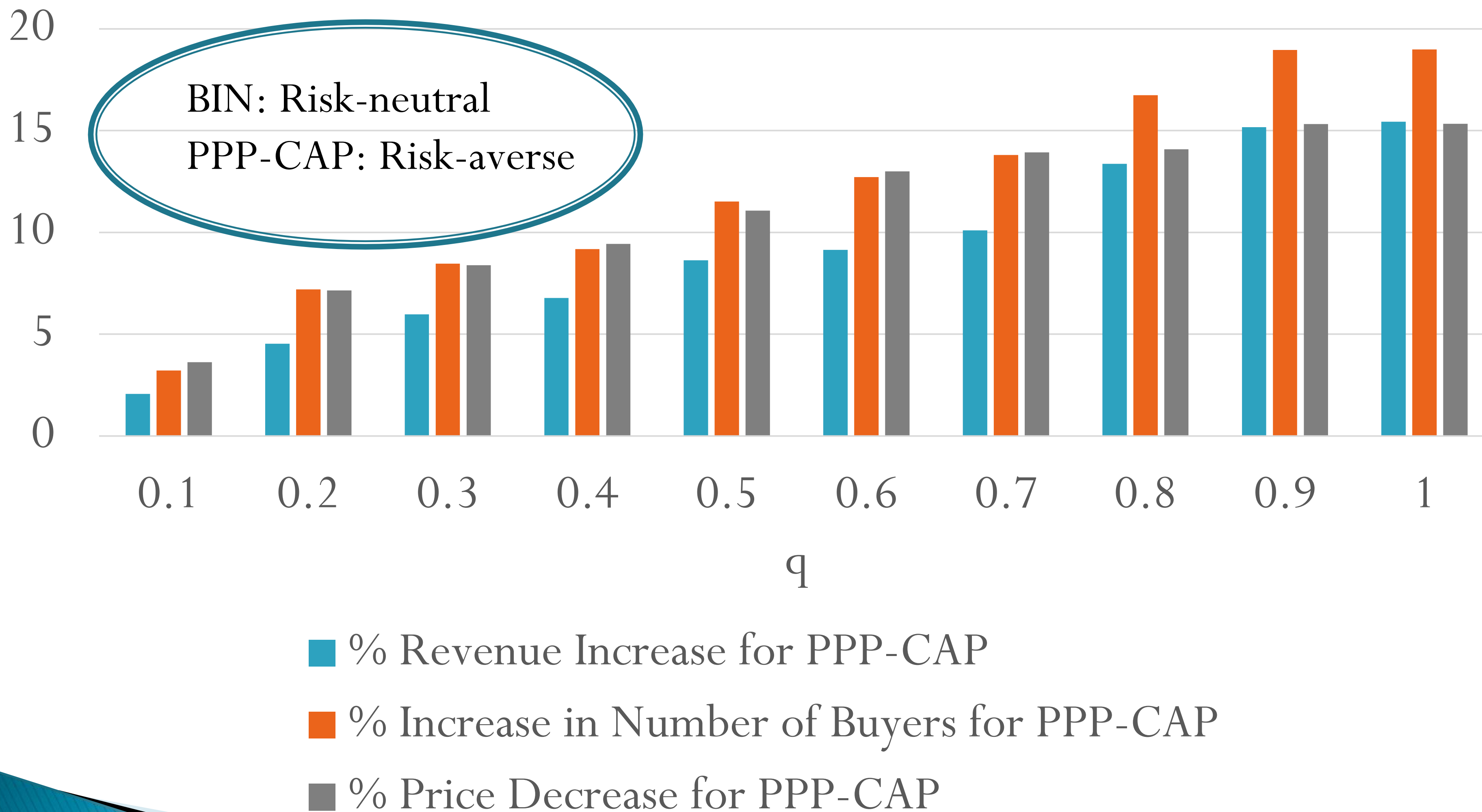
PPP-CAP obtains higher revenue, yields higher buyer utility, and completely removes buyer risk

- Reduce the PPP price continuously till PPP revenue = BIN revenue
  - PPP's social welfare increases beyond BIN's; but revenue matches
  - Social welfare = Buyer utility + Seller revenue

Binary value model: PPP-CAP vs BIN, with initial values drawn from Normal distribution truncated in [0,1], (μ=0.2, $\sigma = \mu/c$) E[Time alive] = $(\text{initial-value})^{0.5}$



Binary value model: PPP-CAP vs BIN, with initial values drawn from Normal distribution truncated in [0,1], (μ=0.2, $\sigma = \mu/5$) E[Time alive] = (initial-value)^q

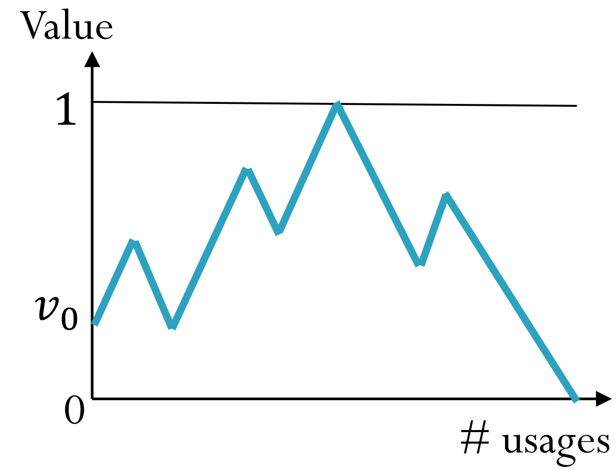


Random walk model

$$V_{t+1} = V_t \pm \Delta_t$$

Buyer's value:

- $V_0 \in [0,1]$ for the first usage
- Evolves as a random walk with step-size δ; i.e., $V_{t+1} = V_t \pm \delta$
- Reflection at 1 and absorption at 0



Buyer knows:

- V_0 and the random walk governing value evolution
- Value for t-th usage only after $t-1$ usages

Seller knows:

- The distribution F from which V_0 is initially drawn
- The random walk governing value evolution

Coming up:

Rev(BIN) vs Rev(PPP) with

a) risk-neutral buyers

b) risk-averse buyers

PPP revenue with risk-neutral buyer

Q: What is the smallest value for which the buyer accepts a price of p?

<u>Answer:</u>

- Certainly for all $v \geq p$, buyer accepts
- But even if $v < p$, buyer could accept, hoping for his value to climb up

Let $U(v, w, p)$ denote the buyer's expected future utility when his:

- current value is v
- price per usage is p
- purchase lasts until his value $> w$

Purchase lasts till value is at least $w^* = argmin_w U(w + \delta, w, p) \geq 0$

When do you stop buying?

- Q: Suppose your value is 0.2, and the per-round PPP price is 0.5; Random-walk of step size $\delta = 0.01$. What will you do?

  a) Buy

  b) Reject ?

  c) Indifferent ?

  ?

- At a price of ½, the buyer never stops buying until his value hits 0
  - Even a buyer with value δ, still buys at a price of $1/2$

PPP revenue with risk-neutral buyer

- Recall: $w^* = argmin_w \, U(w+\delta, w, p) \geq 0$
- Calculations: When $p \leq \frac{1}{2}$, we have $U(w+\delta, w, p) \geq 0$ for all $w \geq 0$

$\Rightarrow$ At a price of ½, the buyer never stops buying till his value hits 0

- Even a buyer with value δ, still buys at a price of $\frac{1}{2}$

$\Rightarrow$ Revenue of PPP is at least half the cumulative value of buyer

- $R_0^{PPP} \geq \frac{1}{2} \cdot C(v)$

PPP results in near perfect price discrimination for risk-neutral buyers

What about risk-averse buyers?

Intuition: PPP gets better as the buyer becomes more risk-averse

1. Rev(PPP) $\geq \Theta(\frac{1}{\delta})$ Rev(BIN) for risk-averse buyers
2. PPP also offers much larger buyer utility than BIN
3. The factor $\Theta(\frac{1}{\delta})$ is tight

In between risk-neutral and risk-averse

Risk-neutral buyer:

- Buy whenever expected utility ≥ 0

Risk-averse buyer:

- Buy only when $P[\text{Utility} < 0] = 0$

α-Risk-averse buyer:

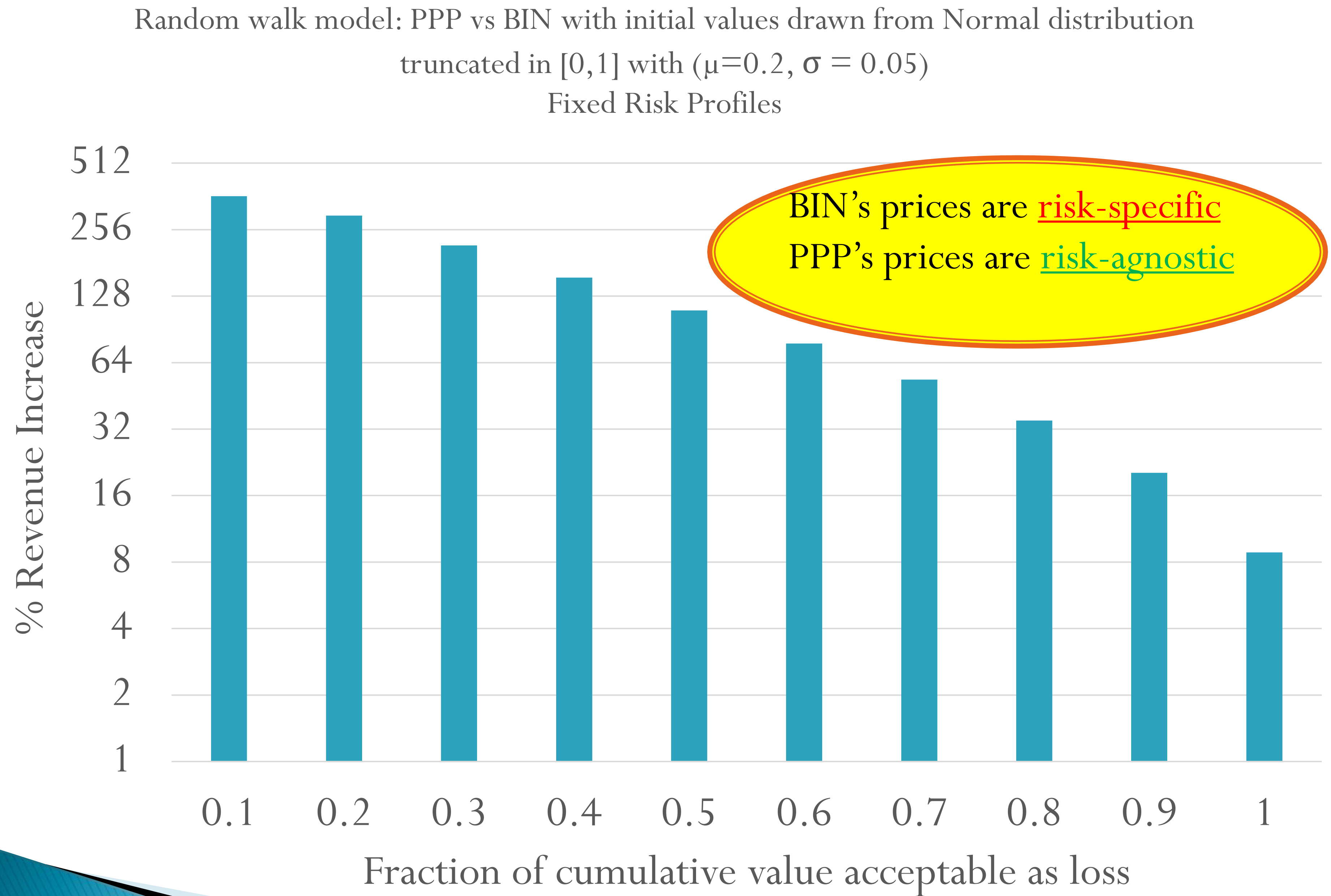
- Buy only when
  - Expected utility ≥ 0, and, $P[\text{Utility} < -\frac{1}{\alpha}] = 0$

Risk-neutral: $\alpha = 0$

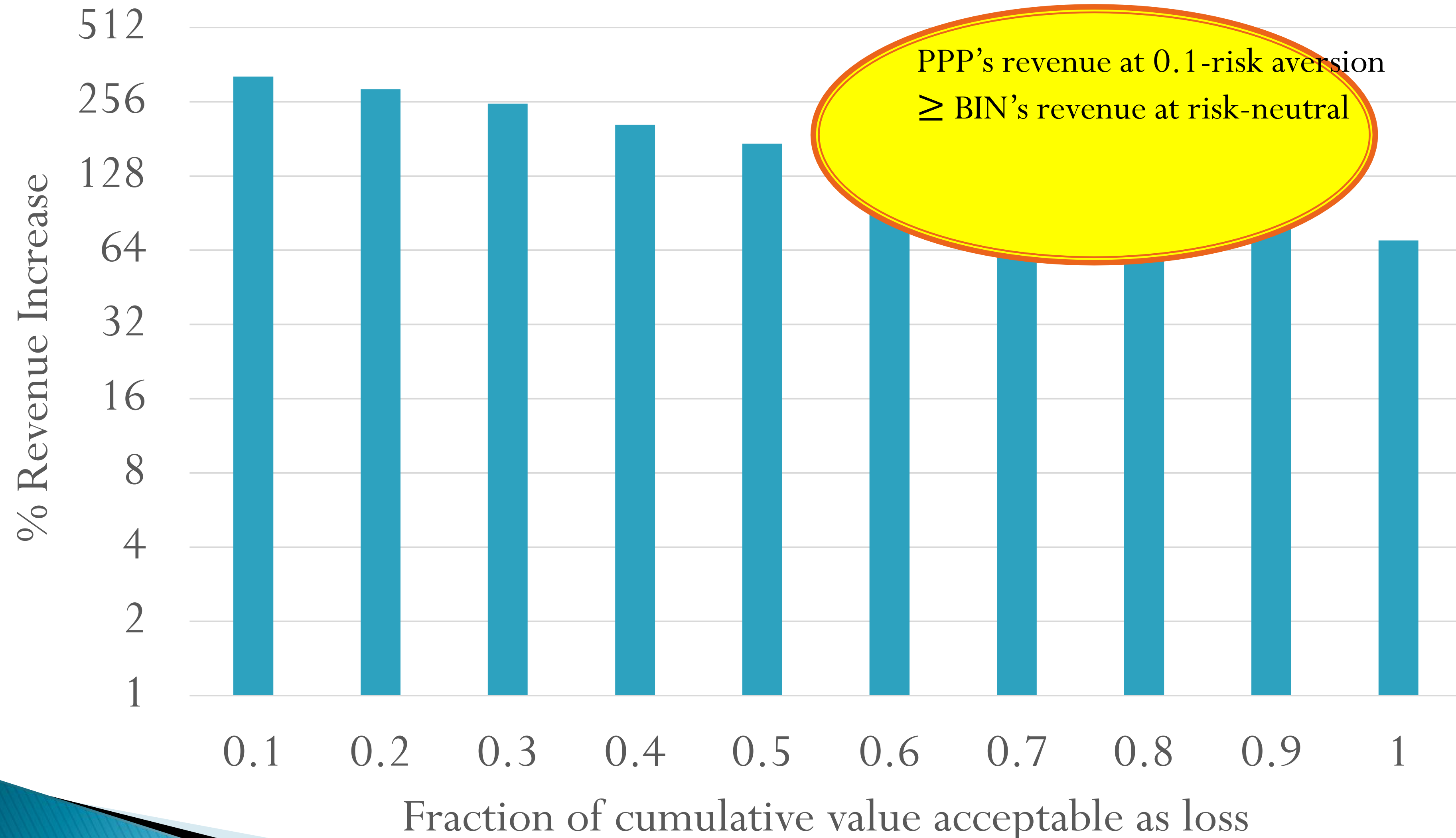
Risk-averse: $\alpha = \infty$

Theorem: For every α, and for every distribution of initial values, there exists a PPP scheme with

$$R_\alpha^{PPP} \geq \frac{1}{32} R_\alpha^{BIN}$$


Random walk model: PPP vs BIN with initial values drawn from Normal distribution truncated in [0,1] with (μ=0.2, σ = 0.05)
Fixed Risk Profiles
512
256
128
64
32
16
8
4
2
1
% Revenue Increase
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Fraction of cumulative value acceptable as loss
BIN's prices are risk-specific
PPP's prices are risk-agnostic

Random walk model: PPP vs BIN with initial values drawn from Normal distribution truncated in [0,1] with (μ=0.2, σ = 0.05)
Bayesian Risk Profiles with Truncated Normal distribution in [0,1], σ = 0.3



Can we get a constant fraction of cumulative value even with risk-averse buyers?

Free trial

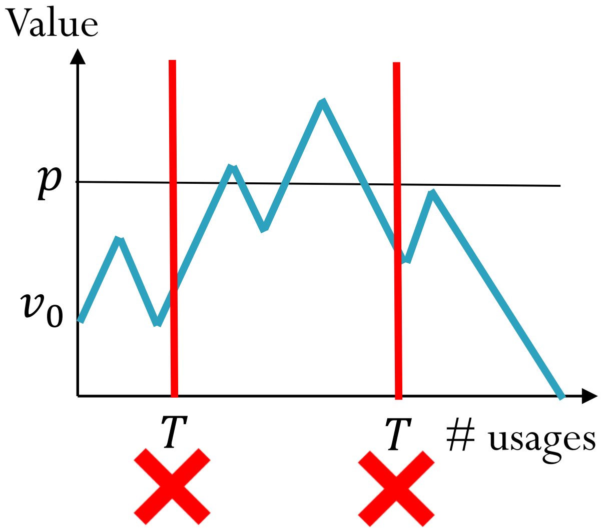
- Offer the product free for the first T usages
- Charge a price of p per usage there after

Risk-averse buyer behavior: Buy only if current value $\geq p$

Hope: Buyer's value will climb sufficiently high during the free trial period

Q: How to set the # of free trials T and price p

- T large enough for value to hit p
- T small enough so that the random walk spends sufficient time above p



Free trial

Q: What is the expected time for buyer's value to hit 1, given that it hits 1?

- $E_v[\tau \mid v_\tau = 1] =$?

Lemma (Levin+Peres+Wilmer'09): $E_v[\tau \mid v_\tau = 1] = \frac{1-v^2}{3\delta^2}$

$$\leq \frac{1}{3\delta^2}$$

Free trial

Let $T = \frac{2}{3\delta^2}$ be the number of free trials

Markov's inequality: $P_v[\tau > T \mid V_\tau = 1] \leq \frac{1}{2}$

Let T' be the number of rounds after free trial is over for which value is $\geq p$

- $Rev(PPP) = R_\infty^{PPP} = p.\mathbf{E}_v[T']$
- $\mathbf{E}_v[T'] \geq \mathbf{P}_v[\mathrm{V}_\tau = 1] \cdot \mathbf{P}_v[\tau \leq T \mid V_\tau = 1].(h_{1p} - T)$

$$\geq v \cdot \frac{1}{2} \cdot \left[\frac{(1-p)^2 - \frac{2}{3}}{\delta^2}\right]$$

- $\Rightarrow R_\infty^{PPP} = p \cdot v \cdot \frac{1}{2} \cdot \left[\frac{(1-p)^2 - \frac{2}{3}}{\delta^2}\right]$
- For sufficiently small p, $R_\infty^{PPP} = \Theta\left(\frac{v}{\delta^2}\right) = \Theta(C(v))$

Free trial + PPP results in near perfect price discrimination for infinitely risk-averse buyers

Summary

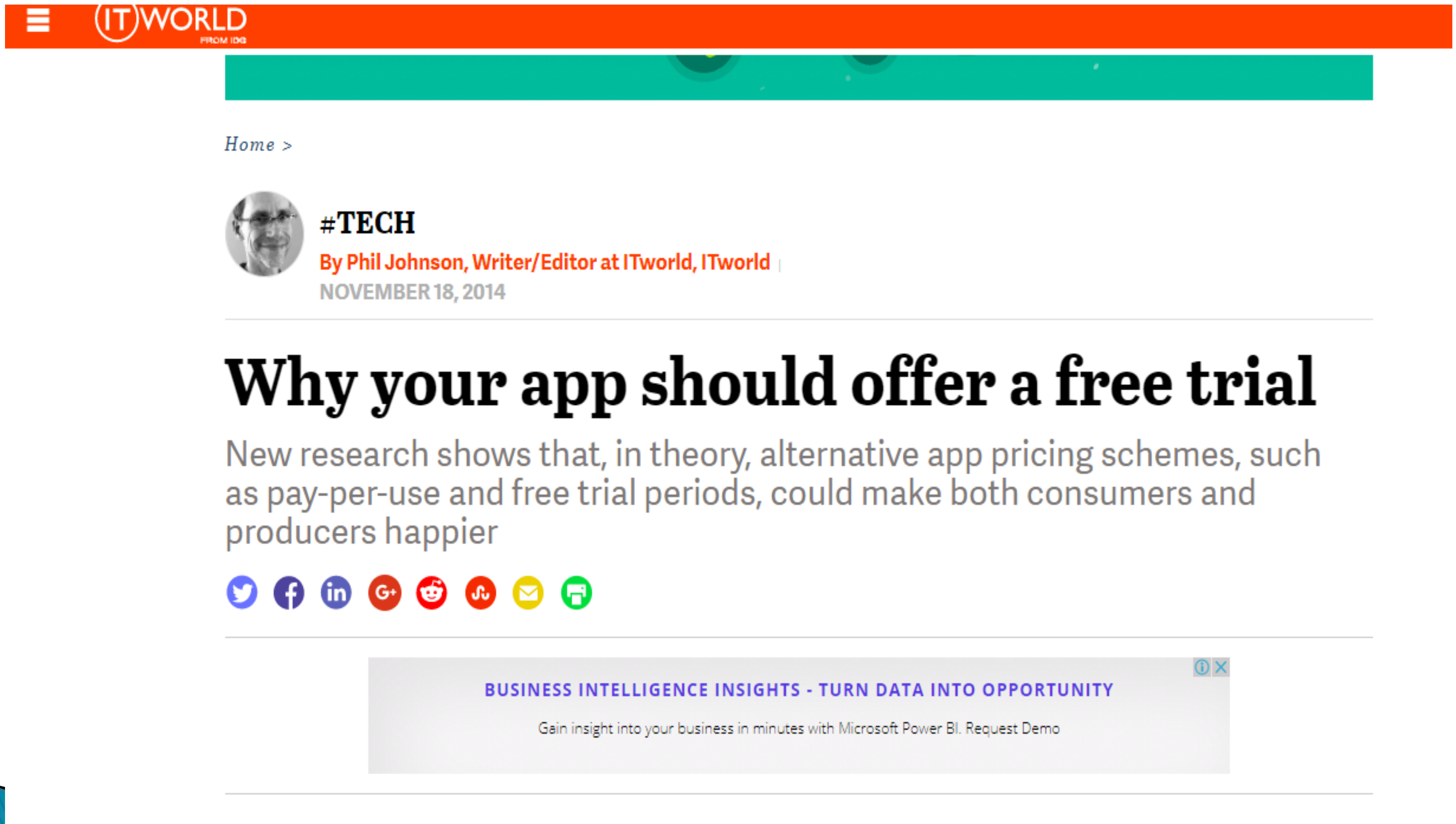
- Usage-based value evolution creates opportunities for alternate payment schemes
- Pay-Per-Play schemes provide substantial advantages over the traditional Buy-It-Now scheme in terms of
  - Revenue
  - Buyer utility
  - Eliminating risk
- Free trial for a few rounds combined with PPP results in near perfect price discrimination even for infinitely risk averse buyer

Summary

- All the results extend to general martingale value evolution
- Buyer need not know anything about the value evolution: just buy when value exceeds price
- Seller need not know about distribution of buyer's value evolution. Just a few conservative estimates

Impact

1. Research featured in IT-world article
2. An app-maker tried the PPP scheme for his app after seeing the article!



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#TECH

By Phil Johnson, Writer/Editor at ITworld, ITworld

NOVEMBER 18, 2014

Why your app should offer a free trial

New research shows that, in theory, alternative app pricing schemes, such as pay-per-use and free trial periods, could make both consumers and producers happier

Open questions

1. Try other value evolution models: super-martingale seems the most realistic for value evolution over time.

2. What are the strategic aspects of offering a PPP scheme?
   - E.g. The music streaming industry has converged to a $9.99 per month model (Xbox music, Google music, Spotify, Deezer,…)
   - If one of them shifts to a PPP scheme, capped at $12, what are the strategic aspects of such a move?

3. What are natural experiments to answer questions like:
   - Does a music pass offering a pay-per-play subscription, capped at $12, increase or decrease revenue?
   - By how much?
   - How many new subscribers will such a modified plan bring?

Plan for the talk

Mechanisms for repeated interactions between seller and buyer (eg. Internet ad auctions)

- Review a few strands of literature
  - Buyers with *independent* values over time (additive)
  - Buyers with values *evolving* over time (additive)
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- Discuss main results & commonly used techniques
- Present future directions / open problems

The repeated sales setting

- A single seller offering a fresh copy of an item every day for n days
- To the same buyer, with additive valuations
  - Buyer's <u>private</u> value v remains the same every day
  - Private value v initially drawn from a <u>publicly known distribution</u> F
  - Seller's cost normalized to 0

<u>Rules of the game:</u>

Seller: Can post a price every day

Buyer: Take-it-or-leave-it at the posted price

Fishmonger's problem*

One day interaction

Seller

$p_F^* = \frac{1}{2}$

Accept/Reject

Buyer

$v \sim F = U[0,1]$

Revenue $= p_F \cdot P[v \geq p_F] = \frac{1}{4}$

* Thanks to Amos Fiat for suggesting the name for this problem.

Fishmonger's problem*

Two days interaction

Seller

Buyer

$v \sim F = U[0,1]$

Same v for both rounds

Revenue = ?

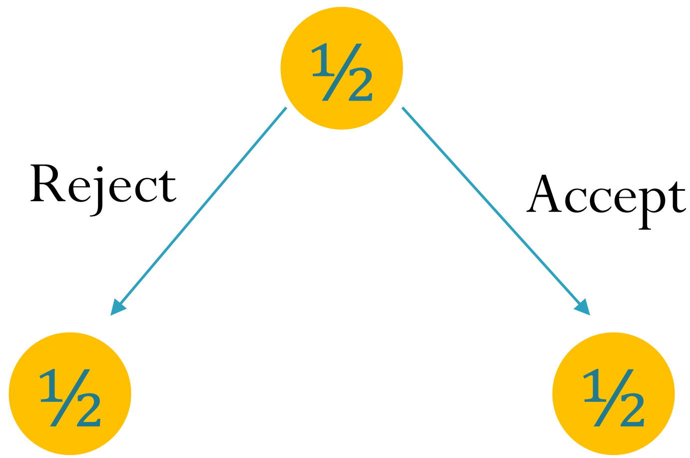
* Thanks to Amos Fiat for suggesting the name for this problem.

Game tree

If the seller could commit to future prices:

Day 1:

Day 2:



Revenue for 2 days $= 2 \cdot \frac{1}{4} = \frac{1}{2}$

Revenue for n days $= n \cdot \frac{1}{4} = \frac{n}{4}$ (Myerson optimal revenue)

What if the seller cannot commit to future prices?

No commitment from seller

Q: If the seller cannot commit to future prices:

- a) seller extracts almost entire value of buyer ?
- b) seller gets exactly Myerson optimal revenue $(n/4)$?
- c) seller doesn't even get Myerson optimal revenue ? ✓

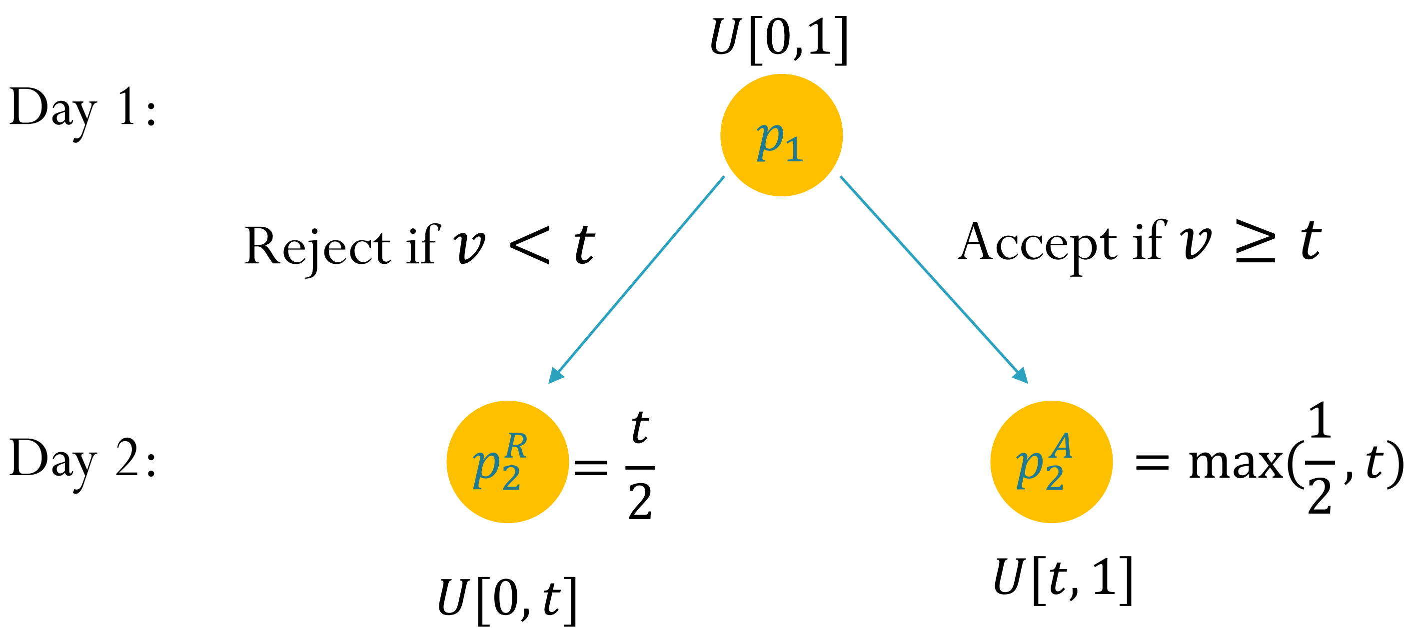
Proof by contradiction: Here is a single-round mechanism with more than Myerson's revenue

1. Solicit buyer's value
2. Simulate repeated mechanism and pick 1 round uniformly at random. Allocation and pricing are decided based on that day
3. Buyer's utility matches the one in equilibrium, so he reports true val

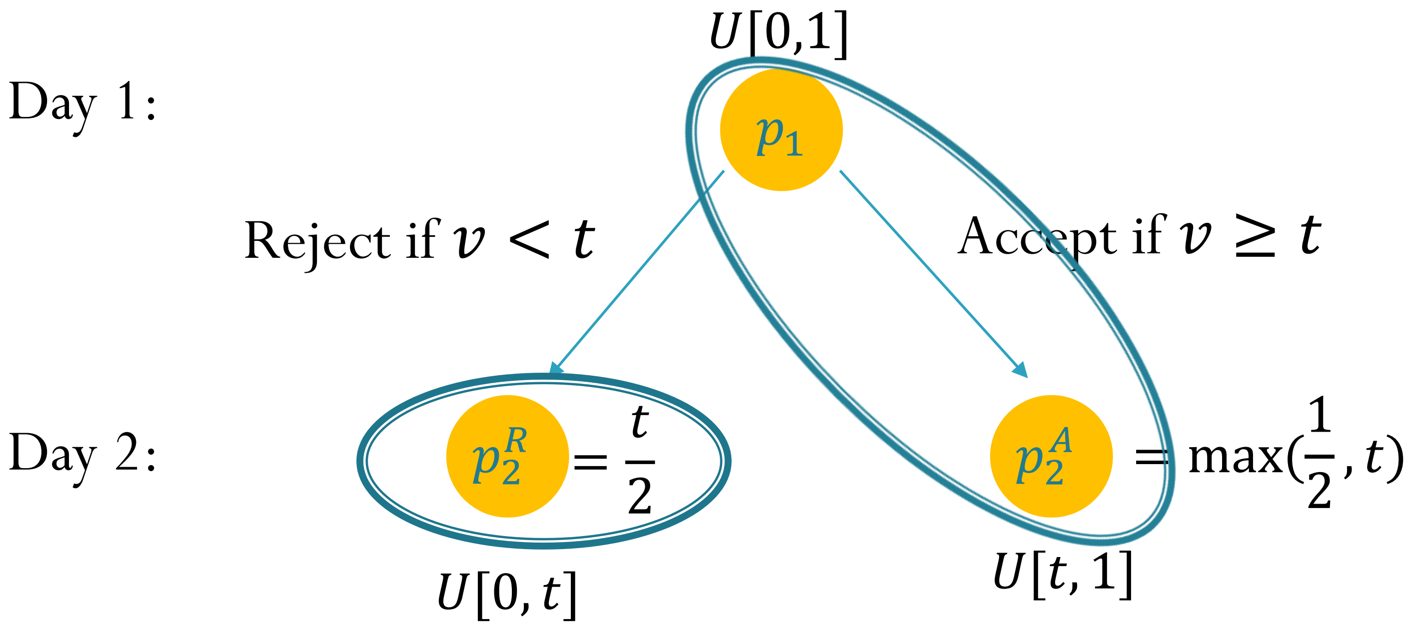
For many distributions, the revenue doesn't even grow with n

Game tree

Seller <u>cannot</u> commit to future prices:



Indifference at threshold



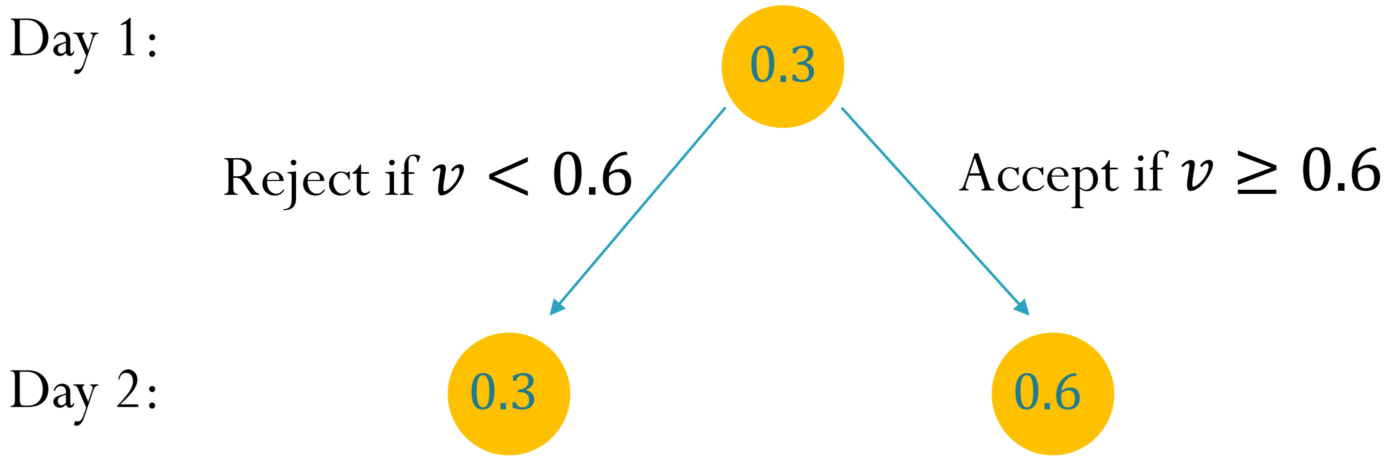
Buyer with value $v = t$ should be indifferent between accepting and rejecting $\Rightarrow$

$$t - \frac{t}{2} = t - p_1$$

$$\Rightarrow p_1 = \frac{t}{2}$$

$$\text{Revenue} = (1-t) \cdot \left[\frac{t}{2} + \max\left(\frac{1}{2}, t\right)\right] + \frac{t}{2} \cdot \frac{t}{2}$$ → maximized at $t = \frac{3}{5} = 0.6$

Solution for 2 rounds



Revenue $= \frac{9}{20}$ → 10% smaller than Myerson optimal revenue of $\frac{1}{2}$

Things change from 3 rounds onwards…

Lack of commitment & Perfect Bayesian Eq. (PBE)

Perfection: Strategies are in equilibrium for every sub-game

Bayesian: Seller does a Bayesian update of his beliefs about buyer's distribution

Formally, a seller's strategy specifies:

1) A price p_1 to be posted in 1st round
2) For every possible price $x \in [0,1]$ in 1st round,
   - a 2nd round price p_2^R if buyer rejects in 1st round the price of x
   - a 2nd round price p_2^A if buyer accepts in 1st round the price of x

Formally, a buyer's strategy specifies:

For every possible value, history of prices and accept/reject decisions, and every possible current price x, whether accept or reject

Previous work

Hart & Tirole [1988]:

- Finite horizon, n rounds
- 2 point distribution $v \in \{ l, h \}$

$\forall$ except the last few (i.e. $O(1)$) rounds, price $= l$

Even if the buyer and seller discount future utilities by $1 - \delta$

- Really bad deal for the seller
- Unnatural and not really seen in practice. Why?

Schmidt [1993]:

- Discrete distributions
- $l =$ lowest point in the support

$\forall$ except the last few (i.e. $O(1)$) rounds, price $= l$

No threshold equilibria

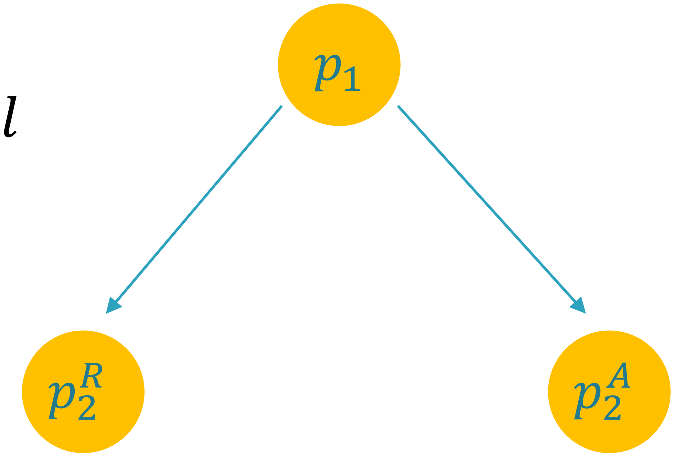
Devanur+Peres+Sivan'15:

Possible explanation for why we don't see the eq. in practice

- Posit: "Threshold Equilibria" = natural equilibria
  - Otherwise, seller's belief supported on fragmented intervals
  - E.g. $U[0, \frac{1}{5}]$ w.p. $\frac{1}{2}$, and $U[\frac{1}{3}, \frac{2}{3}]$ w.p. $\frac{1}{2}$

- Characterize when threshold eq. exists:
  - Only for those distributions where $p_1 = l$ in a two rounds game

- Almost never?



1-sided commitment to rescue?

Devanur+Peres+Sivan'15:

1-sided commitment from seller: cannot increase price, can decrease price

- ≡ price guarantee
- Not decreasing the price is harder to enforce
- Decreasing price is beneficial to both buyer & seller.

Results

- Unique threshold equilibrium with some restrictions
- For $U[0,1]$, revenue is $\sqrt{\frac{n}{2} + \frac{\log n}{8} + O(1)}$

Multiple buyers to rescue?

Immorlica+Lucier+Pountourakis+Taggart'17:

- Study the same problem (seller cannot commit), but there are n buyers
  - Seller posts a single price each day
  - If more than one buyer interested in buying, allocate uniformly at random

- There exists a unique PBE after refinements:
  - Where seller gets a constant fraction of Myerson's revenue

- PBE structure: explore + exploit
  - Slowly raise price; keep raising if at least two buyer are interested
  - After that, post the highest price that the remaining buyer is guaranteed to buy

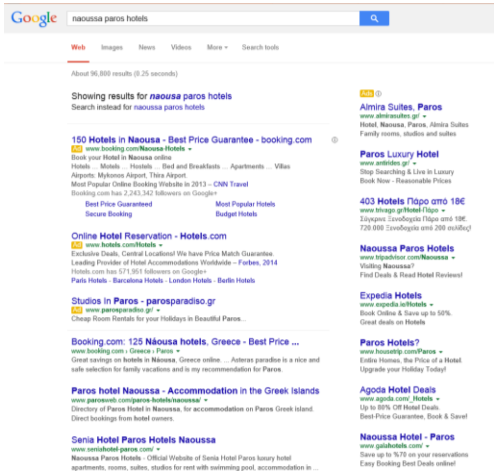
Future directions

Build upon this to handle more general settings

- Multiple buyers, multiple sellers, multiple items, auctions, etc.
- Both seller and buyer have private information

Motivation:

- Behavior based price discrimination
  - privacy issues related to tracking
  - Loyalty cards, cookies, etc.



Plan for the talk

Mechanisms for repeated interactions between seller and buyer (eg. Internet ad auctions)

- Review a few strands of literature
  - Buyers with *independent* values over time (additive)
  - Buyers with values *evolving* over time (additive)
  - Buyers with *fixed value* over time (additive)
  - Buyers with *fixed value, but unit-demand / fixed budget*, and unknown supply
- Discuss main results & commonly used techniques
- Present future directions / open problems

Multi-parameter auctions with unknown online supply

Devanur + Sivan + Syrgkanis'18:

- Two unit-demand buyers
- Two different items (say, watch and sunglass)
- 4 private values: $v_{11}, v_{12}, v_{21}, v_{22}$
- Watch arrives on day-1
- Unknown: whether or not sunglass will arrive on day-2

- Allocation has to be made immediately when item arrives
- Pricing can be done at the end of 2 days
- Design auctions to maximize welfare

Take two out of three

Problem captures three crucial aspects

- Multi-parameter agents
- Online arrival of items
- Need a truthful mechanism

When any one of these constraints is dropped, problem becomes trivial

Take two out of three

Multi-parameter + online + tr~~uthful~~

- [Feldman+Korula+Mirrokni+Muthukrishnan+Pal'09]: simple greedy algorithm based on marginal valuation gives a ½ approximation

Take two out of three

Multi-parameter + ~~online + ~~truthful

- VCG is optimal

Take two out of three

~~Multi-parameter +~~ online + truthful

- Run second-price auction each day
- On the last day, if k items have arrived, charge everyone the $k+1$-th highest price

Even if we wanted prompt pricing (can't wait until last day):

- Babaioff+Blumrosen+Roth'09: $O(\log n)$ approximation, where n is number of bidders

Attempt all three: get nothing

Multi-parameter + online + truthful

Devanur + Sivan + Syrgkanis'18:

- No deterministic auction can get *any finite* approximation to welfare

1. What about randomized mechanisms?
   - There's a trivial $\min(m, n)$ approximation, where m is number of items and n is number of agents. Anything better possible?

2. What about Bayesian valuations?

3. What happens when arrival is stochastic?
   - Eg. When second item arrives with probability p, we can implement VCG with truthful-in-expectation guarantee.
   - Can we generalize to arbitrary number of items?