Locating and Sizing Plants for Bottling Propane in South India

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Shri Shakti LPG Ltd. (SSLPG) imports and markets propane (referred to as liquefied petroleum gas (LPG) in India) in south India. It sells LPG in packed (cylinder) form to domestic customers and commercial establishments through a network of dealers. Dealers replenish their stocks of filled cylinders from bottling plants, which in turn receive LPG in bulk from the cheaper of SSLPG's two import-and-storage facilities that are located on the Indian coast. We implemented integer programming to help SSLPG decide on the locations and long-run sizes of its bottling plants. We estimate that our recommended configuration of bottling plants is about $1 million cheaper annually than the one that SSLPG had initially planned.

The Shri Shakti Group is a 20-year-old group of companies based in Hyderabad, India. It operates primarily in the areas of education, hospitality, and propane manufacture and distribution. (In the rest of the paper, we refer to propane as liquefied petroleum gas (LPG) as is done in India. Further, in India, LPG is usually a mixture of propane and butane.)

Until recently, only government oil companies (GOCs) distributed LPG in India in both forms, bulk and packed (in cylinders). Since 1993, the Government of India has permitted the private sector to import LPG and market it in both bulk and packed forms at free market prices.

The flagship company of the Shakti group, Shri Shakti LPG Ltd. (SSLPG), is one of the first private-sector companies in India to avail itself of the new opportuni-

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ties afforded by the Indian government's decision. SSLPG is implementing a Rs. 5 billion integrated LPG project covering the import, storage, bottling, and distribution of LPG in cylinders and in bulk to industrial, commercial, and domestic customers all over south India. (As of April 1997, one US dollar was about 36 rupees.) The project also includes manufacturing state-of-the-art valves, pressure regulators, and cylinders under foreign collaboration.

In rural areas, SSLPG could in principle use mobile bottling plants.

SSLPG has acquired 200 acres of land for setting up bulk LPG storage and its own import facilities at Kakinada port (located on the eastern coast of south India), both of which it has commissioned. Also, M/s Asia LPG Pvt. Ltd. (a wholly owned subsidiary of SSLPG) has submitted the highest quotation to the New Mangalore Port Trust for leasing 20 acres of land for 30 years in the vicinity of Mangalore port (located on the western coast of south India) for importing LPG. Thus, SSLPG envisages an import-and-storage facility at Mangalore in addition to the one at Kakinada.

SSLPG will sell LPG in bulk to industrial customers directly from the two storage facilities. It markets LPG in packed form to domestic customers and commercial establishments through its network of dealers (also called gas agencies). When a customer's cylinder runs out of LPG, the dealer replaces the empty cylinder with a filled cylinder at the customer's location (dealer and customer are typically at most five miles apart). The dealer recovers the cost of transporting cylinders from the commission it receives on a per-refill basis from SSLPG.

Each dealer town (town in which SSLPG has one or more gas agencies) is associated with a bottling plant. Each dealer sends the empty cylinders in full truckloads to its associated bottling plant. Since the plant maintains an inventory of filled cylinders, the trucks are fully loaded with filled cylinders on their return trips to the dealer.

Each bottling plant receives LPG in tankers from the cheaper of the two import-and-storage facilities (those at Kakinada and Mangalore)—cheaper in terms of the per-ton cost of procurement and transportation. The tankers are dedicated to transporting LPG, and hence, SSLPG pays the transporters for both delivery and return trips to the storage facilities.

Especially in rural areas, SSLPG could in principle use mobile bottling plants. Such a plant is mounted on a skid and placed on the chassis of a truck. The truck then moves with an LPG tanker to the various dealers. At each dealer location, the plant is connected to the tanker while it fills the empty cylinders.

Alternatively, SSLPG could install a bottling pump in each dealer town. LPG tankers could then travel periodically to the various LPG bottling pumps to fill the empty cylinders in the respective dealer towns.

Both of these logistical strategies are cost effective; for instance, they minimize the to-and-fro transportation of cylinders, which is costlier on a per-ton-of-LPG basis than the to-and-fro movement of LPG tankers, and they require less capital investment partly because expensive fire-fighting equipment is mandatory only at locations where LPG is stored overnight. However, in the interests of public safety, the chief controller of explosives has not given the requisite approval, which seemingly can be obtained only through coordinated efforts by the entire Indian LPG industry.

SSLPG is developing an extensive network of dealers in south India, which is strongest in the state of Andhra Pradesh (which includes both Hyderabad and Kakinada). SSLPG markets LPG to both domestic and commercial customers in cylinders of 15 kg capacity. A typical domestic customer consumes eight refills per year (120 kg of LPG per year) while a typical commercial customer consumes about 108 refills per year. Thus, SSLPG treats a commercial customer as being equivalent to 13.5 domestic customers. It targets 1.224 million domestic-equivalent LPG connections by 1998. This translates to a target of about 147,000 metric tons per annum (MTPA) and to an annual turnover of packed sales alone of about 50 million dollars.

> Private-sectors marketers are not playing on a level field with the government oil companies.

To meet this sales projection, SSLPG has already commissioned an LPG bottling plant of 35,000 MTPA (metric tons per annum) at Kakinada and another plant of 12,000 MTPA at Hyderabad. Further, Asia LPG is operating a 12,000 MTPA plant near Bangalore (a major metropolis in Karnataka, a state neighboring Andhra Pradesh). SSLPG has provided for the future expansion of capacity at the Kakinada plant to up to 65,000 MTPA.

To satisfy the residual sales potential, SSLPG has to build additional bottling plants. While facility decisions are intrinsically strategic in character, a peculiar feature of LPG bottling plants renders these decisions even more so. SSLPG cannot increase the capacity of a bottling plant unless it provides for such an increase in the original layout and correspondingly incurs a substantial portion of the cost of expansion during the original construction itself. Failing such a provision, SSLPG will have to shut down the plant and rebuild it to expand its size, because safety distances, fire-fighting facilities, and so forth depend on plant size.

The GOCs have an existing market and a waiting list of customers that is reportedly nearly 10 times the number of fresh connections that are issued annually. Hence, even if a GOC builds a large plant, it can hope to utilize 90 percent or more of the built-in capacity in six months or so.

However, the market for private-sector LPG is new. Further, as D. V. Satya Kumar, the executive director of SSLPG, observes, SSLPG and other private-sector marketers are not playing on a level field with the GOCs; the Indian government subsidizes nearly Rs. 70 of the total cost of about Rs. 160 that the GOCs incur in supplying a refill to the domestic sector. As a result, notwithstanding the necessarily gradual development of the dealer networks of private-sector companies, such as SSLPG, customers' bookings with these companies for connections are growing slowly. The additional capital that is required with each new connection (for procuring valves, regulators, and cylinders) is another reason why SSLPG might take three to four years to attain the target of 1.224 million domestic-equivalent connections.

Plant sizes are always discrete; SSLPG is considering five plant sizes. Owing to the slow growth in bookings, even SSLPG's smallest plant will take almost a year to attain a level of utilization that would warrant upgrading its capacity to the next level. The provision for expansion that SSLPG has to make in the original construction will be a dead investment until the capacity is actually expanded.

The greater the number of cylinders in the system, the greater the cost.

The foregoing discussion implies that SSLPG has to optimally locate bottling plants and determine their long-run sizes as well as time the commencement of operation and expansion of capacity of these plants. In terms of both modeling and availability of data, it is truly formidable to formalize the statement of this problem. The dealer towns number over 400, and the sites for plants, about 20. Besides, we would need to adopt a multi-period framework for modeling the problem. Various sources of uncertainty further complicate the problem, such as the actions of competitors, growth in bookings and in the utilization of the existing plants, and corporate takeovers by SSLPG. (For instance, SSLPG acquired Asia LPG during the course of the study. Further, SSLPG became aware of the likelihood of an import-and-storage facility at Mangalore only two months before our final report was due; as a result, we had to redo all our computations.)

Instead of a multi-period framework for modeling, we adopted a single-period framework in which SSLPG specified a target year. We then determined the configuration of plants to build (specified by location and long-run size) as well as the service area (in the target year) of each of these plants. (The service area of any plant is the subset of dealer towns of SSLPG assigned to it.) Thus, we did not specify when SSLPG had to commission a plant or when it had to increase the capacity of a plant; we only assumed that if a plant was built, then in the target year, its capacity equaled its long-run size. We sought to minimize the total cost of operations in the target year.

It was important that Satya Kumar, the executive director, shared this perspective of the problem with us. He affirmed that once the top management of SSLPG knew what the locations and long-run sizes of bottling plants should be, it would use its judgment to schedule the commissioning and expansion of bottling plants based on such factors as the utilization of existing plants, the extent of the dealer network, and the volume of bookings.

Two studies concern the design of real-world LPG distribution systems. Fölsz, Mészáros, and Rapcsák [1995] described the application of various management science models in the distribution of gas cylinders for a Hungarian company, Prímagáz-Hungária. One of the problems that they considered was the assignment of sales points (dealers) to filling stations (bottling plants). They developed a heuristic for this problem that was based on the solution of a transportation LP. Further, for each filling station, each day, the authors heuristically solved an M-TSP (a traveling salesperson problem with M salespersons) with capacity and time restrictions for delivering cylinders to sales points from the station.

Van Roy [1989] described the use of integer programming in planning the production and distribution of LPG at N. V. Esso Belgium. The distribution system of that company differs from that of SSLPG. First, the company has its own refineries. Second, depots comprise an intermediate level between plants and dealers (who are referred to as transporters). Third, unlike SSLPG, which contracts all of its transportation of LPG in both bulk and packed forms, N. V. Esso Belgium uses its own fleet of trucks, in addition to contract carriers, for transporting LPG between bottling plants, depots, and transporters. Thus, in his optimization model, besides the locations of bottling plants and depots and the assignment of customers to transporters, Van Roy also determined the transportation fleet of the company.

Cost Factors

In operating any plant, SSLPG incurs a fixed annual cost that does not depend on the actual throughput volume of the plant. The capital cost that SSLPG incurs for constructing any plant includes the costs of land, development, buildings, and machinery. SSLPG is raising the required capital through term loans from banks and financial corporations and through investments by promoters. Each plant's annual fixed cost consists of the annual cost of servicing the required capital, salaries and wages, administrative expenses, and corporate office expenses.

Besides the fixed cost of operating plants, the total annual cost of operations also includes the cost of satisfying the annual requirements of all the dealer towns. This cost has three physically evident components: the cost of transporting LPG in bulk to each plant from the cheaper of the two ports; the cost of bottling LPG at the plants; and the cost of transporting cylinders to and fro between bottling plants and dealer towns.

Through feedback from the top management of SSLPG on the results of the model, we discovered other cost factors subsequently in the study. A few months after the study began, we ran a preliminary version of the model in which, following the suggestion of SSLPG, we restricted ourselves to dealer towns and plant sites within the state of Andhra Pradesh. Those dealer towns made up only about two-thirds of the set of towns that SSLPG eventually planned for the state. Through some freak error at SSLPG, we received grossly erroneous estimates of the annual fixed costs for the various plant sizes. As a result, the model recommended only a few large plants whose service areas spanned dealer towns that were more than 500 km away.

Such large service areas prompted Satya Kumar to observe that the greater the distance between a dealer town and its associated plant, the greater the number of cylinders in transit between the two, and the greater the safety stock of cylinders needed at the dealer town. Further, the greater the number of cylinders in the system, the greater the capital cost. Hence, we included the annualized capital cost of cylinders in the cost function.

Restricting the analysis to the state of Andhra Pradesh in the preliminary run masked an important cost factor, which we discovered much later when we solved the model with the entire set of dealer towns that SSLPG had planned for south India. This time, the model recommended that a few plants should be built near state borders and should serve towns in more than one state. In fact, it recommended that one such plant be located in one state but serve mainly towns in a neighboring state. The top management of SSLPG was quick to point out that if it followed the recommendations of the model, it would not be able to take advantage of incentives state governments offered in the form of sales tax exemption or deferment.

To elaborate, if SSLPG built a plant in Andhra Pradesh, it could defer payment for up to 10 years of the state tax on the sales that the plant made to dealers located within Andhra Pradesh. However, the deferred amount could not exceed either 50 or 75 percent (depending on the district of Andhra Pradesh in which SSLPG built the plant) of a certain portion of the capital cost that SSLPG had incurred in building the plant. (Thus, deferring sales tax is roughly equivalent to obtaining an interest-free loan.) Other states in south India offered sales tax exemption (a stronger incentive) for up to 50 percent of the same portion of the capital cost.

It is not difficult to see that because of the time value of money, until SSLPG fully realizes the sales tax incentive on a plant, it may be cost effective for SSLPG to restrict the plant's service area to dealer towns located in the same state.

Besides the sales tax incentive, the management of SSLPG also informed us of a relatively minor cost component that was associated with the bulk transportation of LPG, namely, the cost of loading and unloading LPG that is incurred when the plant is located away from the sourcing port.

The Model

SSLPG's distribution system comprises four levels: ports; bottling plants; dealers; and end users (for example, residential households and commercial establishments). The to-and-fro transportation of cylinders between dealers and end users lies outside the scope of our study because SSLPG pays each dealer a fixed commission per refill. Also, because the quantity of LPG that SSLPG can import at either of the two ports is practically unlimited, each bottling plant receives all of its annual requirement of LPG from the cheaper of the two ports. Thus, in configuring SSLPG's distribution system, we can treat it as comprising just two levels: bottling plants and dealers (or equivalently, dealer towns).

This problem of optimally designing a two-level distribution system is a generalization of the classical capacitated facility-location problem (CFLP) in that, besides locating plants, we also need to specify their (long-run) sizes. We present the formal integer programming model in the appendix. Each decision variable is either of the form $y(i,k)$, which is 1 if a plant of (long-run) size k is built at site i and 0 otherwise, or of the form $x(i,k,j)$, which is 1 if dealer town j is assigned to a plant of size k at site i and 0 otherwise. The objective is to minimize the total annual cost of operations in a target year. The first set of constraints specifies that each dealer town is assigned to a unique bottling plant. The second set of constraints ensures that the capacity of none of the bottling plants is exceeded. The third set of constraints ensures that at most one plant will be built at any site. The fourth set of constraints is logical and ensures that if a dealer town is assigned to a bottling plant of a given size at a specified site, then that plant should be open. These constraints are optional, but we included them because they are very useful in solving capacitated facility-location problems by the LP-based branch-and-bound technique (see for example, Nemhauser and Wolsey [1987, p. 15]).

As one referee pointed out, strictly speaking, our problem is a generalization of the single-source CFLP, that is, every dealer town has to receive all of its requirement of LPG from exactly one bottling plant. For that reason, we should enforce the integrality of the $x(i,k,j)$ variables. However, from the perspective of solving the model optimally, it is extremely difficult to enforce such restrictions and therefore we relax them. (Christofides and Beasley [1983], among other authors, made similar observations about the CFLP.) Besides, the $y(i,k)$ variables were the ones of principal importance. Nevertheless, without exception, in all our runs of the model, the $x(i,k,j)$ variables for fewer than two percent of the dealer towns were fractional.

To reduce the size of the problem, we stipulated a boundary around each site such that only dealer towns within the boundary could be assigned to a plant at the site. (The boundaries for the various sites overlapped, but the model assigned each dealer town to exactly one of those sites whose boundaries contained the town.) Further, for each site, we imposed an upper limit on the size of a plant that could be built at the site.

This is how we defined the boundary for sites other than Kakinada and Mangalore (the ports). For each site, we found the cheaper of Mangalore and Kakinada ports in terms of sourcing LPG. Let $r_2 > r_1 > 0$. The boundary around each site was shaped like a cam and was based on the notion of the Archimedean spiral (Figure 1). The distance from the site to the boundary was shortest (of length r_1) in the direction of the sourcing port and longest (of length r_2) in the opposite direction. The distance from the site to the boundary in any other direction depended linearly on the angle ($\leq 180°$) that was contained between that direction and the direction of the sourcing port.

To justify the above definition of the boundary, we will let DT denote a dealer town that is directed from the site at an angle of $\theta°$ ($0 \leq \theta \leq 180°$) with the direction of the sourcing port (see Figure 1). When θ is zero, LPG travels in bulk form from the port to the site and, after being bottled at the site, travels in packed form to the dealer town, DT, in exactly the reverse direction. However, when θ equals 180°, LPG travels in bulk form from the port to the site and after being bottled at the site, continues in the same direction in packed form to DT. Thus, the smaller the value of θ, the greater the backtracking in

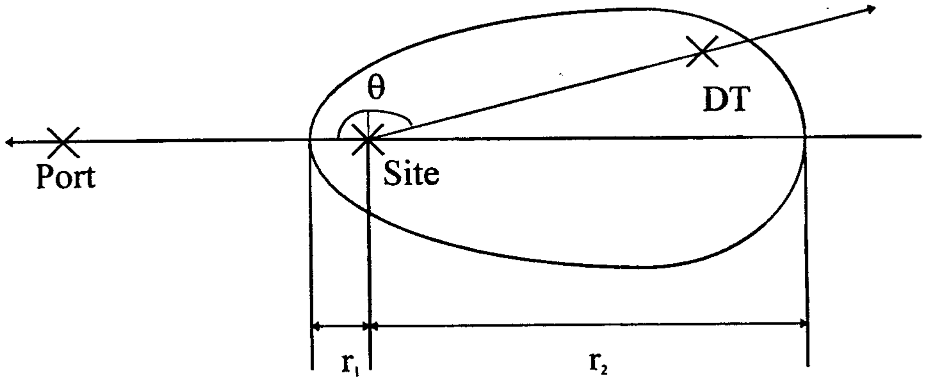

Figure 1: Each site can serve only dealer towns within its boundary, which is closest in the direction of the sourcing port and farthest away in the opposite direction.

the movement of LPG if DT is assigned to a plant at the site. Since such backtracking is in a sense inefficient, the boundary is shortest when θ is 0° and longest when θ is 180°. For a highly simplified representation of this problem that involves square service areas, Geoffrion [1979] analytically showed that optimally each plant will be displaced from the center of its square service area in the direction of the source (which is the sourcing port in our case).

For each of the two sites, Mangalore and Kakinada, the bottling plant is colocated with the sourcing port. Hence, for each of these sites, the boundary extends equally in all directions, that is, it becomes a circle that is centered at the site. In due course, SSLPG will set up import-and-storage facilities at these sites. The cost of bulk transportation and of bulk handling of LPG associated with these two sites will then be zero. Therefore, we made the boundaries for them very liberal. The radius of the boundary for Mangalore was the greater of the two because the unit cost of procuring LPG is greater at Kakinada than at Mangalore.

For each site, we specified r_1 and r_2 as well as the upper limit on plant size on the basis of several factors, such as SSLPG's subjective assessment of the site (especially, the growth potential of SSLPG's sales in the vicinity of the site) and the proximity of the site to either of the ports. Most important, we defined the boundaries considerably more liberally than what SSLPG regarded as being realistic in practice (because, for example, it is hazardous to transport filled LPG cylinders over long distances), and we disallowed only those site-size combinations that SSLPG's top management never seriously considered.

Once we specified the boundary for a site, we computed the sum of the sales potentials (viz., requirements projected in the target year) of all the dealer towns within the boundary. We also used this sum in determining the upper limit on plant size at the site; for example, we would not consider a size of 35,000 MTPA for a site unless the total sales potential within the boundary of the site was at least 31,500 MTPA (90 percent of 35,000 MTPA).

Data

As is well known, facility location models require a lot of data. SSLPG specified the set of dealer towns in each of the four south Indian states as well as the plant sites and the various plant sizes. It also provided us with estimates of the annual fixed cost of operation for each of the plants, the tariff rates for bulk and cylinder transportation, and the unit cost of loading and unloading LPG in bulk. The top management of SSLPG also gave us a formula for estimating the market reach of the company in each of the dealer towns; for each dealer town, the market reach depended on the state to which it belonged, its proximity to Kakinada or Mangalore (whichever SSLPG deemed appropriate), and its proximity to the plants of competitors. Finally, the general manager of finance supplied us with the cash-flow pattern that would ensue if a plant of a given size were built at a given site and if the plant operated at full capacity and served only dealer towns that were within the same state.

The most important data elements that we had to generate were the sales potentials of SSLPG in the dealer towns. Initially, we considered estimating them by using the waiting lists of GOCs, but we subsequently abandoned the idea for a variety of reasons. First, these data are highly confidential and therefore hard to obtain from GOCs. Second, GOCs issue additional dealerships on the basis of the waiting lists that the dealers report; hence, a GOC dealer typically understates his or her waiting list so that the GOC will not be inclined to issue additional dealerships in the sales territory of the dealer that would rob him or her of customers on the waiting list. (In fact, GOC dealers have been known to discourage prospective customers from booking with them for this reason.) Third, many potential customers balk from booking with GOC dealers because of the long waiting times and therefore do not get reported. Fourth, many of SSLPG's dealer towns are in areas that have not been reached by the GOCs.

Therefore, we employed a causal method (regression) for estimating the sales potentials. For reasons of confidentiality, we note only that for each dealer town, we used 1991 census data on the number of residential households in the town to estimate both the total annual domestic sales potential (cumulated over all companies, including the GOCs and private-sector marketers, such as SSLPG) and the annual domestic sales of GOCs in the town. We then computed the difference between the two and applied the market-reach factor to this figure to estimate the annual domestic sales potential of SSLPG in the town. Subsequently, we estimated the ratio of commercial to domestic sales for each dealer town by treating it as a concave, increasing function of the number of residential households. Finally, for each dealer town, we used the corresponding commercial-to-domestic ratio to estimate the total annual sales potential of SSLPG in the town.

Our manner of estimating the ratios of commercial to domestic sales had two implications: (1) the ratio was larger for larger towns; (2) if A, B, and C were three dealer towns such that the number of residential households of B was the average of the numbers of residential households of

A and C, then the ratio for town B would have been higher than the average of the ratios for the other two towns. Both implications are intuitively justifiable.

The model was large primarily because of the large number of dealer towns.

We are currently undertaking a follow-up study jointly for SSLPG and a Fortune-500 US corporation. In that study, we are developing a method for estimating the commercial sales potential of SSLPG in any dealer town independent of domestic sales.

Working independently of us, a marketing analyst of SSLPG also estimated the domestic sales potentials of the company in its dealer towns in the state of Andhra Pradesh. She based her estimates on extensive feedback from SSLPG's dealers in the state. Interestingly, our estimate of the total sales potential of SSLPG in the state was within four percent of hers.

Therefore, for each of SSLPG's dealer towns, we did not alter our estimate of the company's domestic sales potential. However, by varying our estimates of the ratios of commercial to domestic sales in the dealer towns, we generated different sets of estimates of the sales potentials of SSLPG.

To compute the cost of bulk and cylinder transportation that would result for each combination of site, size, and dealer town, we needed both the distance between the site and its sourcing port and the distance between the site and the dealer town. Since all the sites and both the ports were in major cities, we could obtain the actual road distances for all the site-port combinations from road maps. However, most of the dealer towns were small, and we had to approximate the distances between the various sites and these towns by applying suitable multiplicative factors to the corresponding Euclidean (as-the-bird-flies) distances. We found these multiplicative factors, which we specified by site, through regression.

In both bulk and packed forms, LPG is transported only in single-pickup and single-delivery trips of the contractors' vehicles. Therefore, it is easy to accurately (that is, without any approximation) compute the costs of bulk and cylinder transportation of LPG. We included these costs, as well as the cost of bottling LPG, in the objective function coefficients of the $x(i,k,j)$ variables (the $c(i,k,j)$'s).

To model the annualized capital cost of LPG cylinders, we employed a notion that is commonly used in LPG marketing: the ratio of cylinders to domestic-equivalent customers. While the ratio is obviously greater than unity, Satya Kumar suggested that we could assume that for customers in a given dealer town, it varied linearly with the distance between the town and the plant that served the town. Then, to ascertain the annualized capital cost of cylinders that would accrue if a given dealer town were assigned to a specified plant, we merely had to represent the sales potential of the town in terms of the number of domestic-equivalent customers and multiply this figure by the appropriate cylinder-customer ratio and the annualized capital cost of a single cylinder. Thus, the annualized capital cost of cylinders was also reflected in the $c(i,k,j)$ coefficients.

Our manner of incorporating this cost component is somewhat similar to that used by Gelb and Khumawala [1984] in modeling the cost of the potential sales of insurance policies that were foregone when salespersons traveled long distances to make sales calls.

We found it more difficult to accurately represent the savings from sales tax deferments and exemptions in the model. These deferments and exemptions are akin to capital costs in that they are plant specific and are not perennial. However, the capital cost of a plant, by definition, is independent of the volume of LPG that is actually bottled at the plant. In contrast, owing to the time value of money, until the incentive from sales tax deferment or exemption for a plant is fully realized, the net present value of the incentive does depend on the volume of LPG that is bottled at the plant and sold to dealers (in cylinder form) within the same state as the plant.

If we had considered a multi-period model in which we minimized the total cost, discounted to the present, it might have been possible to model the savings accurately. However, such a model would have been very difficult to solve, and moreover, we learned of these savings very late in the study. Therefore, we adopted the following approach. For each plant, we found the annualized cash inflow that would result from sales tax deferment or exemption if in every year, the plant operated to full capacity and sold all the LPG to dealers within the same state. Then, by assuming that the annualized savings in the target year for any plant would be realized in proportion to the volume of LPG that was bottled at the plant and sold within the same state during the target year, we incorporated the savings as negative terms in the $c(i,k,j)$ coefficients.

Solution of the Model

Researchers have published several specialized algorithms for the classical capacitated facility-location problem and its variants. These employ Lagrangian relaxation [Christofides and Beasley 1983] and cross-decomposition [Van Roy 1986], among other techniques. Sridharan [1995] surveys algorithms and heuristics for the capacitated facility-location problem.

However, for various reasons, we chose LP-based branch-and-bound as the basic solution approach. As we had anticipated and as is often the case with OR/MS applications, we became aware of several features of the problem (for instance, major cost factors) only in the course of the study and not at the outset. Moreover, Satya Kumar viewed the present study as the initiation of a model-based approach that SSLPG would use to plan and review its facility decisions in the future. In that light, we believed it was extremely important for us to adopt a robust modeling and solution strategy from the outset. The above-mentioned specialized algorithms depend on the exploitation of the structure of the family of capacitated facility-location models for their success. The addition of a single side constraint can disrupt that structure. In contrast, an off-the-shelf LP-based branch-and-bound software package can handle any objective function or constraint as long as it is linear. Further, while we had ready access to the LINDO software for mathematical programming [Schrage 1989] and its associated object files, we were not aware of any specialized code for our model that was available in the public domain.

The model was challenging to solve because of its size. While the number of $y(i,k)$ variables was 30 to 40, the constraints numbered about 2,500. The model was large primarily because of the large number of dealer towns.

We adopted a novel and indirect method for solving the full-scale version of the problem [Sankaran 1996]. We first reduced the size of the problem by agglomerating subsets of customers into macro-customers. Then, we solved the reduced version by LP-based branch-and-bound, which was enhanced by the automatic generation and inclusion of logically deducible and valid inequalities up front.

The optimal solution to the reduced version yielded us a good incumbent (which was almost always optimal) for the full-scale version. We solved the full-scale version in a similar manner to the reduced version, the difference being that we now had an incumbent.

We implemented our indirect approach by compiling routines in Fortran and linking them with LINDO. Corresponding to each of the different sets of estimates of the sales potentials of SSLPG that we mentioned earlier, we solved the model for six scenarios. Each scenario corresponded to a pair of plant sizes for Kakinada and Mangalore. We solved each of the six scenarios by fixing the two corresponding $y(i,k)$ variables to unity. SSLPG considered this scenario-based analysis to be very appropriate because the cost of bulk transportation and of bulk handling associated with plants at Kakinada and Mangalore will be zero.

We based our recommendations to SSLPG on the commonalities of the optimal configurations for the various runs of the model.

Recommendations to Management

Our recommendations to the top management of SSLPG are confidential because of the competitive nature of private-sector LPG marketing in India. Hence, rather than discuss them at length, we describe their implications for SSLPG's facilities planning.

Even in the course of the study, some members of the board of directors, including Satya Kumar, began to have serious reservations about the configuration that SSLPG had originally envisioned. This comprised three 45,000 MTPA plants at Hyderabad, Kakinada, and Bangalore. SSLPG had planned this configuration at a time when, like other new entrants to the field of private-sector LPG marketing, it was very buoyant about rapidly capturing a major portion of the roughly three million domestic south Indian customers who had been put on waiting lists by the GOCs. However, that did not materialize, primarily because the government continued the subsidy for domestic consumption and because of the wait-and-see behavior of domestic customers, which SSLPG and other private-sector marketers of LPG had not anticipated. SSLPG also had to rethink its facilities strategy because it was awarded the lease for land at Mangalore port, an event which it first envisaged less than two months before our study was to be completed.

In the main, our recommendations agreed with the alternative plans for bottling plants that Satya Kumar and others had begun to consider. What was more important, our study helped resolve the issue because we compared our proposed configuration with the original configuration (which we modified to reflect Mangalore as an import facility and to accommodate our estimate of SSLPG's total sales potential in the target year). We ascertained the cost of this (suitably modified) original configuration by fixing those $y(i,k)$ variables to unity that corresponded to plants that were opened in the configuration and fixing the remaining $y(i,k)$ variables to zero, and then optimizing the resulting LP over the $x(i,k,j)$ variables (in effect solving a transportation LP). Our analysis revealed that the cost in the target year under the proposed configuration was less than that of the original configuration by about $1 million.

Our recommendations also helped to settle a debate between members of the board of directors of SSLPG and other senior officials who differed about what facilities strategy the company should adopt. A major point of difference was the long-run capacity that SSLPG should provide for in the Hyderabad plant. Some of the officials drew upon their prior experience in the GOCs (which have plants of up to 200,000 MTPA capacity) and urged for 45,000 MTPA. Satya Kumar, among others, feared that might be excessive.

However, ultimately SSLPG could acquire only enough land near Hyderabad for a plant of 12,000 MTPA capacity. Our study revealed that this was a fortuitous outcome for SSLPG; in all cases, the model recommended a plant of 12,000 MTPA capacity at Hyderabad.

We also had the opportunity to examine the arguments and counterarguments of key participants in the debate. It was quite apparent to us that they suffered from the drawbacks that typically characterize conventional analyses in strategic distribution planning [Geoffrion 1975, p. 19]. Most important, the arguments were not holistic; they tended to be piecemeal by nature and regional in scope (for example, a state-by-state analysis), and they considered only major dealer towns and a subset of the relevant cost factors. As Satya Kumar noted during a presentation of our methodology and recommendations, our modeling framework adopted a total-system perspective that could overcome the inherent flaws in manual methods.

In fact, the success of OR/MS in this study was in large measure due to the unstinting support of the top management of SSLPG, in particular, Satya Kumar. He had a firm conviction regarding the utility of OR/MS in facilitating strategic decision making in the present context. This was not altogether surprising, because he was quite knowledgeable about the potential of OR/MS, having had a few graduate courses in the area and having published in refereed OR/MS journals!

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APPENDIX

The Model

Notation:

i: index for plant sites.
j: index for dealer towns.
k: index for plant sizes.

Decision variables:

$y(i,k)$: 1 if a plant of size k is built at site i, 0 else.
$x(i,k,j)$: 1 if dealer town j is assigned to a bottling plant of size k at site i during the target year, 0 else.

Parameters:

$SP(j)$: estimated sales potential of SSLPG in dealer town j in the target year.
$C(k)$: capacity of a plant of size k.
$c(i,k,j)$: the cost (during the target year) of assigning dealer town j to a bottling plant of size k at site i.
$F(k)$: annual fixed cost of operating a bottling plant of size k.

Objective function:

$$\text{Min} \Sigma_i \Sigma_k F(k)\, y(i,k) + \Sigma_i \Sigma_k \Sigma_j c(i,k,j)\, x(i,k,j)$$

Constraints:

$$\Sigma_i \Sigma_k x(i,k,j) = 1 \text{ for all } j. \quad (1)$$

$$\Sigma_j SP(j)\, x(i,k,j) \quad (2)$$

$$\leq C(k)\, y(i,k) \text{ for all } i \text{ and } k.$$

$$\Sigma_k y(i,k) \leq 1 \text{ for all } i. \quad (3)$$

$$x(i,k,j) \leq y(i,k) \text{ for all } i, k, \text{ and } j. \quad (4)$$

$y(i,k)$ is 0–1 for all i and k, $x(i,k,j)$ is 0–1 for all i, k, and j.

D. V. Satya Kumar, Executive Director, Shri Shakti LPG Ltd., Begumpet, Hyderabad—500 016, India, writes: "The study on optimal locations and sizes of LPG bottling plants that was conducted on our behalf by Dr Sankaran and Mr Srinivasa Raghavan has been very useful to us in planning our facilities decisions.

"We find the overall recommendations

of the authors quite reasonable, all the more because they are grounded on a comprehensive, model-based analysis of the underlying decision problem. The approach is scientific and the model has addressed the problem as a whole instead of a conventional approach of one to one comparisons which cannot give optimal solutions. The study has helped to settle a prolonged debate within our organisation regarding our strategy for bottling plants. This study would definitely help us in saving our annual operating costs by optimally sizing and locating our bottling plants which we are glad to have undertaken in the planning stage itself. I also foresee that it will be extremely useful to us for renegotiating with the concerned funding agencies on the manner in which we may redeploy the loans towards the construction of bottling plants in a optimal fashion rather than how it was originally conceived."