

## Design and analysis of optimal material distribution policies in flexible manufacturing systems using a single AGV

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Modern automated manufacturing processes employ Automated Guided Vehicle (AGV) for material handling, which serve several machine centres (MC) in a factory. Waiting time for resources such as AGVs are the longest elements that make up the Manufacturing Lead Time (MLT). Hence, optimal scheduling of AGVs can significantly help to increase the efficiency of the manufacturing process by minimizing the idle time of MCs waiting for the raw materials. In this paper, we will analyse the requirements for an optimal schedule and then provide a mathematical framework for an efficient schedule of material delivery by an AGV. The optimal schedule depends on several factors, such as the processing speeds of MCs, the speed and the material carrying capacity of the AGV, and system dependent overheads (such as loading and unloading time, machine set-up time, distance travelled, etc). A mathematical model is developed and then a strategy for optimal material distribution of the available raw material to the MCs is derived. With this model, the optimal number of MCs to be utilized will also be determined. Finally, the material delivery schedule employing multiple journeys to the MCs by the AGV will be carried out. Through rigorous analysis and simulation experiments, we shall show that such a delivery strategy will optimize the overall performance.

### 1. Introduction

The Flexible Manufacturing System (FMS) emerged as a highly competitive manufacturing strategy in the late 20 century (Kaighobadi and Venkatesh 1994, Viswanadham and Narahari 1998) and has been a subject of intense research and exploration. It utilizes computer controlled automation systems to integrate the machine centres (MCs) with the *material handling system* (MHS). Such a system is capable of scheduling the production of a specific part at any of the available MCs, as well as efficient routing of the raw material on the shop floor to minimize the idle time of resources and to maximize the overall efficiency.

Manufacturing lead time (MLT) is the total time required to process the product in the manufacturing plant. In a typical manufacturing environment, MLT is much greater than the Actual Processing Time (APT). The APT, which comprises the set-up time and processing time, could be a small fraction of the MLT. Loading and

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unloading, transport, waiting and queuing times constitute the rest of the MLT (Viswanadham and Narahari 1998), which we will refer to as *overheads*. Significant improvements in manufacturing efficiency can be achieved by utilizing automated MHSs which help in minimizing the idle time of MCs and waiting (queuing) times for transport resources. Material handling techniques have improved significantly over the last several decades and can be found in (Kaighobadi and Venkatesh (1994) and Viswanadham and Narahari (1998)), for example. The intelligent techniques have made the most significant impact on material handling schemes, as these techniques rely on integrated planning in the FMS decision making processes. In addition, the overall plan of manufacturing, referred to as the Integrated Manufacturing Planning Problem (IMPP) in the literature, determines the subsequent scheduling, routing and dispatching decisions. Research presented by Seo and Egbelu (1999) provides a planning model by decomposing the integrated plan into sub-plans for machine selection and operations sequence.

An Automated Guided Vehicle (AGV) system is an intelligent and versatile MHS to transport materials to various locations in a FMS. AGVs are self-propelled driverless vehicles that travel along a passive guided path, connecting remote locations, typically the ware-house, to the MCs. Interchangeable trolleys are also available to transport a variety of payloads and a computer-control mechanism allows for maximum flexibility and the ability to interface with other building systems, such as elevators and automatic doors (Viswanadham and Narahari 1998). These vehicles range from small mail deliverers to large transporters, capable of delivering hundreds of tons of material. Centralized computers control AGV movements and coordinate the deliveries with other material handling devices. The operation of AGV requires controlled material loading and unloading, determination of the AGV path on the shop floor and the guidance mechanism (Hoff and Sarker 1998). The AGV controller makes several decisions on the use and control of the AGVs. These include, (1) dispatching decisions to decide which of the several AGVs to be used to transport raw material to the MCs; (2) sequencing decisions to decide the order of use of AGVs, if several AGVs are used; (3) routing decisions to decide the route to be taken to reach the machines; (4) scheduling decisions to decide the start, wait and finish times to avoid collision and shop locking (deadlock or stalemate) (Reveliotis 2000); and finally, (5) loading decisions to decide on how much material per trip is to be carried for each of the machines.

The motivation for this paper stems from the needs of large-scale manufacturing enterprises, wherein demand for scheduling very large amounts of material among the available machines in a minimum amount of time, is a time-critical objective and also, when the number of AGVs that can be allotted to cater for a particular session is restrictive. For instance, when a large amount of material is to be processed and if there are, say, five machines available at this time instant, then from the material warehouse centre, AGVs must transport the material to the respective machine sites for processing. Our problem context arises in the following real-life scenario wherein, given a set of machines that are capable of processing that particular material and that a single AGV is available (assigned for this production section) for transporting the material from the material warehouse to each of the machine centres, how best we can schedule the entire material among the machines for processing so that the entire processing of the material incurs a minimum amount of time? Thus, since only one AGV is available, we need to consider the dynamics of AGV uploading and transportation mechanisms and also the machine speeds, and set-up time delays in

deciding the amount of material that can be assigned to each machine. Clearly, the above-mentioned context motivates an examination of the problem posed and it is a scheduling problem of practical interest.

### 1.1. *Our contributions*

In this paper, we consider a discrete flexible manufacturing system, capable of producing discrete products, comprising a single AGV and several MCs. The characteristics of the material to be processed are as follows. We first assume that the entire raw material to be processed can be divided into a number of portions and these portions are scheduled on individual machines for processing. Hereafter, we refer to the overall material to be processed simply as *material*. The material, by and large, we envision is of homogeneous type, by which we mean that the end product(s) after processing can be obtained by solely processing the assigned portion of the material in its entirety and it does not need any intermediate processing at other machine sites. In addition, when the material to be processed is partitioned into a finite number of portions, when each portion results in an identical end product, the number of products may be in proportion to the amount of material assigned to a particular machine.

- Based on the material processing characteristics mentioned above, we first formulate the problem as a min-max optimization problem that aims to obtain an optimal material processing time. The model considers some typical parameters that influence the processing time, such as AGV uploading time, transportation delays from the warehouse to the production centres by the AGV, machine start-up delays, and material processing time at the individual machine sites.
- Using the model mentioned above, we design a strategy by which the material is transported to the machine centres for processing, referred to as the *material distribution strategy*. Here we attempt two modes of distribution, namely, single instalment policy (SIP) and multi-instalment policy (MIP), to be described in sections 3 and 4, respectively. We design these strategies of material transportation and processing and analyse their performance in terms of the overall finish times of the entire processing material supplied to the system. In SIP, the material is divided into as many portions as there are number of machines whereas, in MIP, the material is divided into a number of portions greater than the number of machines in the system; however, we attempt to deliver the material to the respective machine centres in more than one instalment in a *round-robin* fashion. This policy, as we will show, will reduce the idle times of the MCs and maximize the utilization of each MC. Since the machines' speeds can be heterogeneous in practice, we consider the fact that the number of instalments for each MC may not be the same. For these policies, we use the mathematical model to determine the optimal quantity of the raw material that the AGV should supply to each MC and the number of machines to be utilized so that the entire raw material supplied to the system is processed in a minimum amount of time.
- We present a discussion on the effect of the material distribution sequence on the processing time. That is, we investigate the material distribution sequence with respect to the processing time of the entire material.

- It may be noted that, in all the above analysis, we have assumed that the material can be arbitrarily divided into any number of portions. Although this assumption eases analytical tractability, in a real-life situation the amount of material that can be assigned to a machine must be such that material assigned must be sufficient to produce at least one product. This means there is a minimum amount of material that should be assigned to produce a single product. We refine our so-far optimal solutions obtained to take into account this practical requirement.

## 1.2. Related work

In this section, we present a brief summary of the related work in this domain that is relevant to the research contributions in this paper. FMS operation has been a significant subject of research in the last two decades and is inherently a complex system of several technology components. It provides challenges in solving constrained optimization problems to the researchers. Various manifestations of exhibility in FMS, especially routing and scheduling have been studied in isolation. General non-standard routing problems in FMS have been described in Suri and Desiraju (1997), Reveliotis (2000), Soylu *et al.* (2000), Tsubone and Hrikawa (1999), Caprihan and Wadhwa (1997), Kats and Levner (1998), Mahmoodi *et al.* (1999), Oboth *et al.* (1999). Standard queuing models seem to be unsuitable to analysing the configuration because there is a need to take into consideration complex issues such as state dependent routing (Suri and Desiraju 1997). These dependencies may generate several states in a queuing theoretical formulation. An iterative queuing network model built around mean value analysis, considering state dependent routing, is also proposed by Suri and Desiraju (1997). Even though the single AGV system may seem to be an oversimplified scenario in an analytical model, it still serves as a meaningful representation for several reasons. It is one of the most common configurations in a manufacturing environment and it simplifies material handling and control mechanisms. Moreover, this simple model may serve as a building block for more complicated configurations involving multiple AGVs (Reveliotis 2000). For configurations utilizing multiple AGVs, the results presented by Reveliotis (2000) discuss the issue of conflict-free routing and deadlock prevention. In Soylu *et al.* (2000), an artificial neural network algorithm to find the shortest route of a single free ranging AGV is presented.

Information about the state of the system is of paramount importance in determining the scheduling policy of the FMS (Piramuthu *et al.* 2000, Chen *et al.* 1999, Saygin and Kilic 1999, Chan 1999). The scheduling policy must adapt to the changing dynamics of the underlying system, such as breakdown of a resource or an overflow of a material holding buffer at a MC (Kim *et al.* 1999). Once the routing and scheduling decisions have been made, the focus shifts to dispatching, sequencing, and loading of the AGV. These concern the selection of specific AGVs and are decisions driven by constraints such as availability and capacity of the AGV (Hoff and Sarbar 1998). Dispatching and sequencing could be random or rule based, as considered by Hoff and Sarbar (1998). Loading of AGV determines the amount of material delivered to a MC. Mathematical modelling of loading policies determines the optimal number of AGVs to be used and the loading of AGV. This affects the variable production cost as discussed in Moon and Hwang (1999), Rajotia *et al.* (1998), Bukchin and Tzur (2000). In this paper, we mathematically model the load-

ing policy so as to minimize the processing time for the entire raw material submitted to the system for production.

## 2. Definitions, mathematical model and assumptions

In this section, we define the terms used in the mathematical model representing the optimal scheduling time. We base our model on a directed flow graph with AGV nodes and processing nodes. The flow graph depicts the causal precedence between events.

### 2.1. Scheduling strategy and some definitions

The scheduling strategy is described as follows. We assume a Flexible Manufacturing System(FMS) consisting of  $N$  heterogeneous machines that can work with different rates of production and have different set-up times. Further, we assume that the system has a single AGV that caters for these  $N$  machines. Figure 1 shows a possible layout of such a system. The AGV allocates portions of the total available raw material to these machines in a particular order, say, from  $M_1$  to  $M_N$ . Let the portion of the material assigned to machine  $M_k$  be denoted as  $\alpha_k$ .

As mentioned in the introduction, we assume that the amount of time that the AGV incurs in uploading the material from the source site, before it starts transporting it to the destination machine site, is directly proportional to the amount of material it allocates to that machine. This is because the AGV incurs time to collect the material to be allocated and also to upload. Note that this may also involve the help of any additional robot-driven mechanisms. This component also involves the internal set-up time delays of the AGV. Typically, this is nothing but the additional overhead components incurred by the AGV before it actually initiates transportation. Further, we denote the actual transit time delay as  $\theta_{agv}$ , and assume that the distances of the destination machine sites from the material centre do not vary and remain unchanged. This is a fairly reasonable assumption as the location of the production centres (machines) will be usually fixed. In real-life situations, the speed of the AGVs can be varied and, hence, the transit time delay to each of the

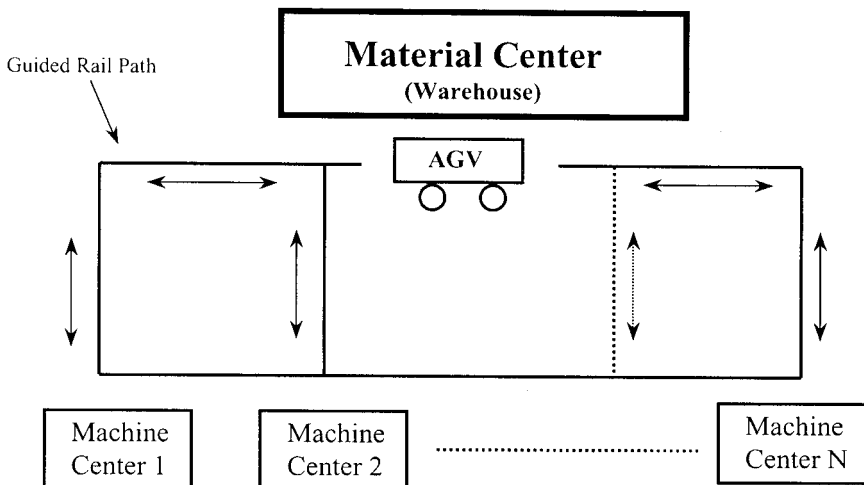


Figure 1. Layout of an FMS with a single AGV serving  $N$  machine centres.

machine centres from the source site can be made more-or-less identical, and hence, we assume that  $\theta_{\text{agv}}$  is a constant. In practice, this quantity captures an average transit time delay. Upon downloading the material at the processing machine site, the machine takes a finite (non-zero) amount of time to start the processing. Without loss of generality, we assume that this start-up time delay is a constant, denoted as  $\theta_{\text{mac}}$ . Thus, our problem lies in precisely determining the optimal size of the amount of material that has to be allocated to each machine such that the total available material is processed in the minimum possible time. The constraints to this problem are the set-up times, heterogeneous processing speeds of the machines, and the start-up delays, which are assumed to be arbitrary.

Intuitively, the optimal solution or the optimal schedule to this problem is such that all the machines must stop processing at the same time instant. To obtain the optimal solution to this problem, we assume that the total available raw material can be divided arbitrarily into any number smaller amounts or fractions. Note that this assumption is useful in determining the optimal sizes of the portions of the amount to be allocated to each machine. However, it will be made clear later that neither this formulation nor the underlying assumptions will be restrictive, even if the material division demands integer amounts to be scheduled onto each machine. This means that, if the available material to be processed is in terms of some fundamental quantity, then the allocated amount to each machine will be in terms of integral multiples of such a fundamental quantity. This is similar to the case when the material is, say, expressed in terms of number of pieces of certain size and that each machine is allocated  $k$  number of such pieces.

We now introduce some useful notations, definitions and terminology that will be used throughout the paper. We refer to the amount of raw material to be processed as ‘load’, hereafter.

- $\alpha_i \in R^+$  Amount of raw material scheduled on machine  $M_i$ .
- $S_i \in R^+$  The inverse of the processing rate of the machine  $M_i$ , expressed in seconds per unit raw material.
- $\theta_{\text{mac}} \in R^+$  An additive set-up time delay(overhead) component that includes the sum of all delays associated with the processing at the machine site.
- $C \in R^+$  The time taken by the AGV to collect and upload a unit amount of material before the initiation of the transportation process.
- $\theta_{\text{agv}} \in R^+$  An additive constant transit time delay associated with the transportation of the intended material by the AGV to the machine site.

Thus, using the above notations, we see that the transportation time of a material  $\alpha_i$  by the AGV is given by  $\alpha_i C + \theta_{\text{agv}}$  and the total processing time of this material by the machine  $M_i$  is given by,  $\alpha_i S_i + \theta_{\text{mac}}$ . It may be noted that since we assume a single AGV to cater the distribution process, the amount of time taken to deliver the material to the first machine site will be  $\alpha_1 C + \theta_{\text{agv}}$ ; however, from the subsequent deliveries, we will have a time of  $\alpha_i C + 2\theta_{\text{agv}}$ , since the AGV has first to reach the material centre, and then to transport to other machines at processing centres. Thus, it incurs twice the transit delay time. In addition, we assume that  $C < S_i \forall i$ .

## 2.2. Directed flow graph

The material distribution process is described by means of a directed flow graph (DFG) as shown in figure 2. The figure shows two types of nodes. The first level of nodes are referred to as *AGV nodes*, representing the function of AGV in distributing

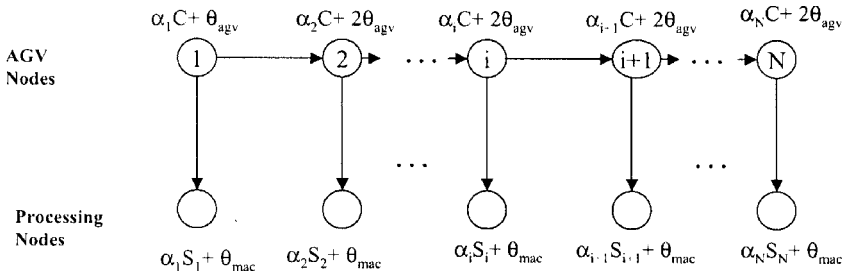


Figure 2. Directed flow graph representation for the material distribution process with  $N$  AGV nodes and  $N$  processing nodes using single-installment policy (SIP).

the material and the other nodes are referred to as *processing nodes*. The weight of a AGV node  $i$  is given by  $\alpha_i C + 2\theta_{agv}$ ;  $i = 2, \dots, N$  (as explained above) and the weight of the processing node  $i$  is given by  $\alpha_i S_i + \theta_{mac}$ ,  $i = 1, \dots, N$ . The directed arrows between the adjacent AGV nodes  $i$  and  $i + 1$  represent the fact that the transportation of the material  $\alpha_{i+1}$  to  $M_{i+1}$  will start only after the transportation of  $\alpha_i$  to  $M_i$  is completed. Similarly, the directed arrows between an AGV node  $i$  and the processing node  $i$  denote that the processing of  $\alpha_i$  by  $M_i$  starts only after receiving the entire material  $\alpha_i$  from the AGV. These directed arrows represent the *causal precedence* relationships between the events. Thus, we see that this directed flow graph represents the material distribution process completely. We use this representation to derive the optimal processing time throughout the paper. We shall now define the following.

- (i) *Feasible material distribution*, denoted as  $\alpha$  is defined as an  $N$ -tuple  $(\alpha_1, \dots, \alpha_N)$  such that  $0 < \alpha_i \leq 1$  and  $\sum_{i=1}^N \alpha_i = 1$ . The equation  $\sum_{i=1}^N \alpha_i = 1$  is referred to as *normalization equation*. Let us denote the space of all possible feasible material distributions as  $\Gamma$ .
- (ii) *Machine finish time* of a machine  $M_i$ , denoted as  $P_i$ , is the time difference between the instant at which the  $i$ th machine stops processing and the time instant at which the AGV initiates the process. This is given by the sum of the weights of the nodes starting from the AGV node 1 until the processing node  $i$ , along the directed arrows. This is referred to as *finish time path*, hereafter.
- (iii) *Material processing time*, denoted as  $T(\alpha, N)$ , is the time at which the entire material is processed, i.e.  $T(\alpha, N) = \max \{T_i(\alpha, N)\}$ ,  $i = 1, 2, \dots, N$ , where  $T_i$  is the finish time of machine  $M_i$ . From figure 2, this is given by the longest finish time path in the directed flow graph and represents the time at which the entire material is processed. This quantity is also referred to as the total lead time (TLT) of the end product.
- (iv) *Optimal material processing time*, denoted as  $T^*(\alpha^*, N)$ , which is the minimum material processing time to finish processing the entire material, i.e.  $T^*(\alpha^*, N) = \min_{\alpha \in \Gamma} \{T(\alpha, N)\}$ .

Thus, from the above definitions (iii) and (iv), we note that our objective is to obtain  $T^*(\alpha^*, N)$  such that,

$$T^*(\alpha^*, N) = \min_{\alpha \in \Gamma} \max_{1 \leq i \leq N} \{T_i(\alpha, N)\}. \tag{1}$$

It is intuitively clear that, in order to achieve optimal material processing time, the feasible material distribution must be such that all the machines must stop processing at the same instant in time. The proof of this assertion can be seen from the fact, that in case the above claim is not true, then this leads to a situation wherein some machines will complete earlier and some others will be working. Clearly, a redistribution can lead to an improved processing time. This fact is also captured in the directed flow graph by equating the finish time paths and solving the recursive equations. We shall use this fact as the optimality criterion to derive the optimal material distribution and analyse the processing time performance of the system.

**3. Closed-form solutions for the optimal material processing time using a single instalment policy**

In this section, we shall derive a closed-form expression for the optimal material processing time by assuming that the sequence of material distribution is from  $M_1$  to  $M_N$  in that order.

*3.1. Analysis using heterogeneous machines*

Consider a system with one AGV and  $N$  heterogeneous machines. There is some material  $L$  to be processed. The material is such that it can be divided into a number of independent portions and delivered to the machines. Our objective in this case is to determine the optimal portions of material that should be delivered to each of the machines so that the entire material is processed in the least possible time. We first consider a case where the number of portions of material is equal to the number of machines and the AGV delivers the entire material to be processed by a machine in a single instalment. From figure 2, we obtain the following recursive equations. The finish time path  $P_1$  of the first machine is given as,

$$P_1 = \alpha_1 C + \theta_{agv} + \alpha_1 S_1 + \theta_{mac}. \tag{2}$$

Similarly, the finish time path  $P_i$  of  $M_i$  is given as,

$$P_i = \sum_{j=1}^i \alpha_j C + (2i - 1)\theta_{agv} + \alpha_i S_i + \theta_{mac}, \quad i = 1, 2, \dots, N. \tag{3}$$

As discussed earlier, for optimality, all the machines must stop processing at the same time. This implies that,

$$P_i = P_{i+1}, \quad \forall i = 1, \dots, N - 1. \tag{4}$$

From equations (3) and (4), we obtain,

$$\alpha_i S_i = \alpha_{i+1}(C + S_{i+1}) + 2\theta_{agv}, \quad \forall i = 1, \dots, N - 1. \tag{5}$$

Similarly,  $P_{N-1} = P_N$  gives,

$$\begin{aligned} & \sum_{j=1}^{N-1} \alpha_j C + (2N - 3)\theta_{\text{agv}} + \alpha_{N-1} S_{N-1} + \theta_{\text{mac}} \\ &= \sum_{j=1}^N \alpha_j C + (2N - 1)\theta_{\text{agv}} + \alpha_N S_N + \theta_{\text{mac}} \\ &\Rightarrow \alpha_{N-1} = \frac{\alpha_N(C + S_N)}{S_{N-1}} + \frac{2\theta_{\text{agv}}}{S_{N-1}}. \end{aligned} \tag{6}$$

From (6) let,

$$\frac{C + S_N}{S_{N-1}} = \beta_N \quad \text{and} \quad \frac{\theta_{\text{agv}}}{S_{N-1}} = \gamma_{N-1}. \tag{7}$$

From (6) and (7) we have,

$$\alpha_{N-1} = \alpha_N \beta_N + 2\gamma_{N-1}. \tag{8}$$

Similarly,

$$\alpha_{N-2} = \alpha_{N-1} \beta_{N-1} + 2\gamma_{N-2}. \tag{9}$$

Substituting (8) in (9) we obtain,

$$\alpha_{N-2} = \alpha_N \beta_N \beta_{N-1} + 2\beta_{N-1} \gamma_{N-1} + 2\gamma_{N-2}. \tag{10}$$

Generalizing (10), we have,

$$\alpha_i = \sum_{j=i+1}^N \beta_j + 2 \sum_{j=i+1}^{N-1} \left( \gamma_j \prod_{k=i+1}^j \beta_k \right) + 2\gamma_i, \quad i = 1, \dots, N - 1. \tag{11}$$

Since the sum of the material distributed to all machines is equal to the total material to be processed, we have,

$$\sum_{i=1}^N \alpha_i = L. \tag{12}$$

Using (12) we obtain,

$$\sum_{i=1}^{N-1} \alpha_N \prod_{j=i+1}^N \beta_j + 2 \sum_{i=1}^{N-1} \left( \sum_{j=i+1}^{N-1} \left( \gamma_j \prod_{k=i+1}^j \beta_k \right) + \gamma_i \right) + \alpha_N = L. \tag{13}$$

Hence,

$$\alpha_N = \frac{L - 2 \sum_{i=1}^{N-1} (\sum_{j=i+1}^{N-1} (\gamma_j \prod_{k=i+1}^j \beta_k) + \gamma_i)}{1 + \sum_{i=1}^{N-1} \prod_{j=i+1}^N \beta_j}. \tag{14}$$

It may be observed from (14) that a necessary and sufficient condition for optimal material distribution utilizing all  $N$  machines exists when,

$$L > 2 \sum_{i=1}^{N-1} \left( \sum_{j=i+1}^{N-1} \left( \gamma_j \prod_{k=i+1}^j \beta_k \right) + \gamma_i \right). \tag{15}$$

Hence, if  $\alpha_N > 0$  then the optimal number of machines is equal to  $N$  and the optimal material distribution is obtained by substituting equation (14) in (11). The optimal material processing time  $T^*(\alpha^*, N)$  is given by,

$$T^*(\alpha^*, N) = \alpha_1(C + S_1) + \theta_{\text{agv}} + \theta_{\text{mac}}, \tag{16}$$

where,  $\alpha_1$  is obtained by using (14) in (11) with  $i = 1$ . It should be noted that given an  $N$ -machine system and all the overhead parameters, it is possible that an optimal processing time may not exist if we attempt to utilize all the machines. This is reflected by the fact that when  $\alpha_N < 0$ , then the optimal processing time is obtained by utilizing fewer than  $N$  machines. Hence, for a given order in which the material is to be distributed, say from machine 1 to machine  $N$ , the optimal number of machines is equal to a maximum value of  $i$  for which  $\alpha_i > 0$ , following that material distribution order.

It may be noted that the above process of determining the qualified set of machines can be iteratively checked. Thus, initially, when there are  $N$  machines available for processing, condition (15) is checked. If it holds then we can utilize all  $N$  machines. However, if it is violated, then we consider utilizing  $N - 1$  machines (preferably dropping the slowest machine) and verify condition (15) for this system. We repeat this process until we obtain a system with the maximum number of machines satisfying condition (15). Our simulation study, in Section 5, indeed identifies the maximal set of machines to be used for a given set of parameters.

3.2. Homogeneous machines

Now, we consider a system of  $N$  homogeneous machines ( $S_i = S, \forall i$ ) served by a single AGV. This case is similar to the above case except that, in this case, the speed of processing of all the machines is the same, i.e.  $S_i = S, \forall i = 1, \dots, N$ . Hence, This implies  $\beta_i = \beta$  and  $\gamma_i = \gamma$ , in (7). From (10) and (11), we have,

$$\alpha_N \left( 1 + \sum_{i=1}^{N-1} \beta^{N-i} \right) + \sum_{i=1}^{N-1} \left( 2\gamma \sum_{j=i+1}^{N-1} \beta^{j-i} + 2\gamma \right) = L. \tag{17}$$

Simplifying (17), we obtain,

$$\alpha_N = \frac{L - 2\gamma(\sum_{i=1}^{N-1} (N - i)\beta^{i-1})}{\frac{\beta^N - 1}{\beta - 1}}. \tag{18}$$

Substituting (18) in (11), we can obtain an optimal material distribution. Denoting the denominator of the right-hand side expression above as  $D$ , the above expression can be further simplified as,

$$\alpha_N = (L/D) + \frac{2\gamma\beta(\beta^{N-1} - 1)}{D(\beta - 1)^2} - 2\gamma \frac{\beta^{N-1}N - 1}{D(\beta - 1)}. \tag{19}$$

Since  $S_i = S$ , for a homogeneous system of machines, we can obtain several interesting results as follows. We can define a polynomial function of degree  $N$ ,

$$g(\beta, N) = (L) + \frac{2\gamma\beta(\beta^{N-1} - 1)}{(\beta - 1)^2} - 2\gamma \frac{\beta^{N-1}N - 1}{(\beta - 1)} \tag{20}$$

such that, for a given number of machines  $N$  and the parameters  $C, S$ , a range of feasible  $\sigma = C/S (= \beta - 1)$  values may be realized for which  $g(\beta, N) > 0$ , thus allowing all the  $N$  machines to be utilized. However, when  $\sigma$  increases, the range of values for which the function  $g(\beta, N) > 0$  will be smaller. This is true since, for a given  $S$ , increasing the value of  $\sigma$  is equivalent to considering large values of  $C$ , which are

nothing but the AGV overheads to collect and upload the material before the initiation of the distribution. Thus, the influence of  $C$  is clearly reflected in our study. Alternatively, one can determine a range of  $\theta_{\text{agv}}$  values for which all the  $N$  machines can be utilized. The range of  $\theta_{\text{agv}}$  for which all the  $N$  machines can be utilized is given by,

$$\theta_{\text{agv}} < \left( \frac{L}{\frac{2(NX - 1)}{(\beta - 1)} - \frac{2\beta(X - 1)}{(\beta - 1)^2}} \right), \quad (21)$$

where  $X = \beta^{N-1}$ . It may be readily verified that the denominator of the right-hand side is a positive quantity. The above equation is a useful relationship in choosing a particular AGV for distributing the material. Given that the uploading time and speeds of the machines almost remain constant, as long as the transit time delay ( $\theta_{\text{agv}}$ ) satisfies (21), all the machines can be involved in processing the load. Thus, in a real-life situation, wherein  $\theta_{\text{agv}}$  can be varied, the above relationship can be used to compare the maximum transit delays of the AGV against the right-hand side expression. Thus, when the worst case delay satisfies the above relationship, all the  $N$  machines can be used for processing the material. It may be noted that one can obtain a closed-form for the processing time. Given the fact that  $\alpha_1$  can be expressed in terms of  $\alpha_N$  (from equations (5) and (18)) and then using (18) and (21) for  $\alpha_N$  and  $\theta_{\text{agv}}$ , we obtain a closed-form expression for processing time.

### 3.3. Effect of sequencing on the processing time

In the previous cases, a closed-form was derived by assuming that the sequence of material distribution is fixed. However, given an  $N$ -machine system and a single AGV, there are  $N!$  material distribution sequences possible. Since the processing speeds of the machines are different, each of these distribution sequences will result in a different total processing time of the material. In this section, we will demonstrate using an example that the total processing time can be altered by changing the sequence of the material distribution and propose a simple greedy algorithm that can fetch a sub-optimal solution.

**Example 1:** Consider a system where a single AGV serves eight machines ( $N = 8$ ). The total material ( $L$ ) to be processed is 10 000 units, the machine set-up delay ( $\theta_{\text{mac}} = 300$  s) and the AGV transit delay ( $\theta_{\text{agv}} = 180$  s). The speed of the machines ( $S_i$  in seconds per unit material) is given as  $S_1 = 8$ ,  $S_2 = 7$ ,  $S_3 = 6$ ,  $S_4 = 5$ ,  $S_5 = 4$ ,  $S_6 = 3$ ,  $S_7 = 2$  and  $S_8 = 1$ , respectively. We first distribute the material in the following order ( $M_1, M_2, M_3, M_4, M_5, M_6, M_7, M_8$ ). Thus, the material is supplied in the increasing order of processing speeds of the machines, starting with the slowest machine. Now, using the closed form solution, the total processing time  $T(\alpha, 8)$  is 13 869 s. It may be noted that, with this sequence, equation (15) is fortunately satisfied, thus allowing all the eight machines to be utilized.

Next, we reverse the order in which we distribute the material to the machines ( $M_8, M_7, M_6, M_5, M_4, M_3, M_2, M_1$ ). Now, in this reversed order, we are delivering the material in decreasing order of the processing speeds of the machines, starting with the fastest machine. We refer to this sequence as a *fast sequence*. We use the closed form solution to determine the optimal processing time. Here, (15) is not satisfied, so we reduce the number of machines utilized to seven eliminating the slowest machine. We observe that (15) is now satisfied with seven machines. In

this case, the total processing time  $T(\alpha, 7)$  is 12 988 s, which is the optimal processing time, the best case. Hence, we conclude that when the machines are served in decreasing order of their processing speeds, it results in the optimal processing time (best case), provided such a sequence exists satisfying (15).

**Greedy algorithm:** When a fast sequence does not exist with all the  $N$  machines, ((15) is not satisfied), we utilize fewer machines. Since the search domain consists of  $\sum_{k=1}^N k!$  sequences, as a greedy approach, we consider a fast sequence employing  $N - 1$  machines, and so on. In such a case, it is always the slowest of the remaining machines that is eliminated. Thus, our search process has only  $O(N)$  complexity to determine a sub-optimal solution.

The following example demonstrates that a solution generated from using a subset of machines using the fast sequence may be better than the solution generated by using all the machines with some other sequence when the fast sequence with all the machines does not exist. This example clarifies the need for the above greedy approach. The numerical values for this example are intentionally chosen to demonstrate this important property.

**Example 2:** Consider a FMS system with  $N = 5$  machines with the following speed parameters:  $\theta_{\text{agv}} = 0.005$ ,  $L = 1.0$ ,  $S_1 = 0.1$ ,  $S_2 = 0.3$ ,  $S_3 = 0.4$ ,  $S_4 = 0.7$  and  $S_5 = 1.0$ . Suppose we choose the fast sequence  $(M_1, M_2, M_3, M_4, M_5)$  to distribute the material, we note that an optimal solution with  $N$  machines using the fast sequence does not exist. We then consider an arbitrary sequence  $(M_3, M_5, M_4, M_1, M_2)$ , we note that an optimal solution exists and it is given by 0.4575. Suppose we choose only  $(N - 1)$  machines using the fast sequence to distribute the load, we observe that the optimal solution exists for this system and is given by  $0.4458 < 0.4575$ . Thus, we observe that by simply attempting to utilize all  $N$  machines using an arbitrary sequence may not fetch a better solution than using a fast sequence with a subset of machines. It would be interesting to attempt a proof of this observation analytically.

#### 4. Multi-instalment material distribution

In the previous section, a closed-form solution for the total processing time was derived by assuming that the AGV delivers the intended portions of the material to the respective machines in one instalment. Thus, in the above policy SIP, the total material is partitioned into as many portions as available machines and AGV delivers material to every MC, one at a time. Further, the AGV returns after every delivery to pick up material for the next MC, and so on. Although this policy is simple, it may result in longer waiting times of the machines. That is, with SIP, first, machines—especially the ones that are served later in the cycle—need to wait for a long time for the material to be delivered to them before they can start processing. Secondly, AGV needs to return to the material centre to pick up material for every machine, causing extra transit delays. Thirdly, a machine receiving its entire portion of processing material in one instalment may not have sufficient storage capacity to hold the portion and may run out of buffer space. These inefficiencies cause idle-times at the machines, resulting in sub-optimal utilization of the AGV, and an increase in the total processing time of the material. Finally, SIP is not suitable if the total material allocated to any machine  $(\alpha_i)$  is greater than the material carrying capacity of the AGV.

To improve the efficiency, we propose an alternative policy that enables the machines to start processing early. This is achieved by reducing their waiting times for the processing material. We call this policy the Multi-Instalment policy (MIP) for material distribution. In this policy, the entire material is divided into, say  $K$  portions, where  $K \geq N$ . The choice of the parameter  $K$  depends on the minimum amount of material required for producing a single product. Further, we assume that the minimum amount of material required to produce a single product is  $\Delta$ , and each of the  $K$  portions is an integral multiple of this fundamental quantity  $\Delta$ , referred to as *granularity*. Without loss of generality, we assume  $\Delta = 1$ . In this policy, the AGV serves the machines in a round-robin fashion as follows. In every trip, the AGV delivers to a machine, a fraction of the total material that is to be processed by that machine. This fraction of the material, delivered to a machine in one instalment is called the grain size. The AGV starts from the material centre with the first instalment of material for every machine. It delivers the first instalment to the respective machines, then returns to the material centre to pick up the second instalment for all machines and so on. The process is repeated until all  $K$  portions are delivered. Note that, in such a policy, it is not necessary that all the machines will receive an equal number of instalments of material, and also the individual grain sizes across instalments and machines are not the same. Such a MIP has two major benefits over SIP discussed in the previous sections. First, the MIP reduces the time a machine has to wait for the material to be delivered to it and hence it can start processing early. Secondly, it reduces the number of trips AGV has to visit the material centre to pick the material, and this reduces the transit delays, which could be a significant overhead in the entire process. Consider a FMS with  $N$  machines with the number of allowed partitions being  $K$ . Our objective is to determine the optimal quantities of material the AGV should deliver in each instalment to every machine such that the total processing time of the entire material is minimized.

#### 4.1. General material distribution equations

As before, we use a directed flow graph (DFG) representation to describe the material distribution process. Let  $\alpha_i$  be the portion to be distributed to a machine. With the above explained material distribution process, the assignment of material portions to the respective portions is as follows. For all  $i \leq N$ ,  $\alpha_i$  is the portion of the material assigned to machine  $M_i$  in the first instalment. For all  $i > N$  and  $i \neq nN$ ,  $n = 1, 2, \dots$ ,  $\alpha_i$  is the material delivered to the  $(i \bmod N)$ th machine in the  $\lceil i/N \rceil$ th instalment. When  $i = nN$ ,  $\alpha_i$  is assigned to machine  $M_N$  in the  $\lceil i/N \rceil$ th instalment.

We use an example to illustrate this policy and derive the material distribution equations below. Consider a FMS with one AGV serving three machines ( $N = 3$ ) using our MIP. Let us assume that the total material is divided into five portions ( $K = 5$ ). The DFG for this process is shown in figure 3. In this case,  $M_1$  is assigned two portions,  $\alpha_1$  and  $\alpha_4$ , in the first and second instalments, respectively.  $M_2$  is also assigned two portions,  $\alpha_2$  and  $\alpha_5$  in the first and the second instalments, respectively. However,  $M_3$  is assigned only one portion  $\alpha_3$  in the first instalment. We denote the finish time path for the  $i$ th machine as  $P_i(q(K))$ , where  $q(K)$  denotes the material distribution with  $K$  portions and is given as  $q(K) = \alpha_1, \alpha_2, \dots, \alpha_K$ . As before, the finish time is optimal when all the machines stop processing at the same time. This implies that the finish time path for all the machines is the same, i.e.  $P_i(q(K)) = P_{i+1}(q(K))$ ,  $i = 1, 2, \dots, K - 1$ . This gives a system of  $K - 1$  equations with  $K$  unknowns. We use the condition that the total material supplied to the

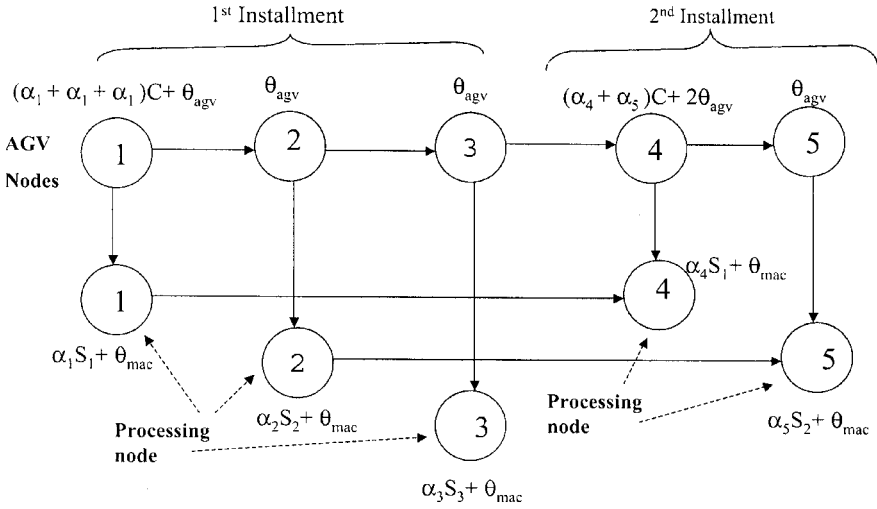


Figure 3. Directed flow graph representation for the material distribution process in two instalments with  $N = 3$  and  $K = 5$  using MIP.

system is  $L$  and solve equations to obtain the optimal material fractions. For ease of computation, we represent the path equations in the form of a  $K \times K$  (in our example, this will be a  $5 \times 5$ ) matrix. We refer to this matrix as the *path matrix*. The finish time path equations for  $P_1(q(5))$  are,

$$P_1(q(5)) = \sum_{i=1}^3 \alpha_i C + \theta_{agv} + \alpha_1 S_1 + \theta_{mac}$$

$$P_2(q(5)) = \sum_{i=1}^3 \alpha_i C + 2\theta_{agv} + \alpha_2 S_2 + \theta_{mac}$$

$$P_3(q(5)) = \sum_{i=1}^3 \alpha_i C + 3\theta_{agv} + \alpha_3 S_3 + \theta_{mac}$$

$$P_4(q(5)) = \sum_{i=1}^5 \alpha_i C + 5\theta_{agv} + \alpha_4 S_1 + \theta_{mac}$$

$$P_5(q(5)) = \sum_{i=1}^5 \alpha_i C + 6\theta_{agv} + \alpha_5 S_2 + \theta_{mac}$$

and, in the matrix form, referred to as the path matrix, the above set of equations can be written as,

$$\begin{pmatrix} (C + S_1) & C & C & 0 & 0 \\ C & (C + S_2) & C & 0 & 0 \\ C & C & (C + S_3) & 0 & 0 \\ C & C & C & (C + S_1) & C \\ C & C & C & C & (C + S_2) \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \end{pmatrix} + \begin{pmatrix} \theta_{agv} + \theta_{mac} \\ 2\theta_{agv} + \theta_{mac} \\ 3\theta_{agv} + \theta_{mac} \\ 5\theta_{agv} + \theta_{mac} \\ 6\theta_{agv} + \theta_{mac} \end{pmatrix}, \tag{22}$$

where the row  $i$  denotes path  $P_i(q(K))$ . The recursive equations generated by performing  $P_i(q(K)) - P_{i+1}(q(K))$  can also be written in a matrix form. We refer to this matrix as the *difference matrix*. From (22), we obtain the difference matrix as follows, where the last row introduced is our normalizing equation  $\sum_{j=1}^K \alpha_j = L$ .

$$\begin{pmatrix} S_1 & -S_2 & 0 & 0 & 0 \\ 0 & S_2 & -S_3 & 0 & 0 \\ 0 & 0 & S_3 & -S_1 & 0 \\ 0 & 0 & 0 & S_1 & -S_2 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \end{pmatrix} = \begin{pmatrix} \theta_{agv} \\ \theta_{agv} \\ 2\theta_{agv} \\ \theta_{agv} \\ L \end{pmatrix}. \tag{23}$$

From (23), we obtain the solution vector as,

$$\begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \end{pmatrix} = \begin{pmatrix} S_1 & -S_2 & 0 & 0 & 0 \\ 0 & S_2 & -S_3 & 0 & 0 \\ 0 & 0 & S_3 & -S_1 & 0 \\ 0 & 0 & 0 & S_1 & -S_2 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}^{-1} \begin{pmatrix} \theta_{agv} \\ \theta_{agv} \\ 2\theta_{agv} \\ \theta_{agv} \\ L \end{pmatrix}. \tag{24}$$

### 5. Discussions of the results

This paper provides a framework to derive the optimal material distribution in a FMS using AGVs. The research contributions of this paper are a logical extension to the related work on FMS process optimization by optimal scheduling and routing of AGVs. The optimal material distribution is a key factor in optimizing the material transportation in a manufacturing environment. We consider a single AGV on the shop floor delivering the processing material to several heterogeneous machines. The objective is to minimize the processing time of the raw material by optimizing the material distribution. This is significant in the domain of FMS as it affects the overall manufacturing efficiency, which in turn affects the product cost. A directed flow graph is proposed to represent the problem mathematically. This representation is elegant in describing all the features of the material distribution strategy. We also take into account all the overheads that penalize the material processing time.

Using this model, we derived a closed form solution. We considered two material distribution schemes, SIP and MIP. The single instalment policy distributes the entire material allocated to a machine in one instalment. The results of simulation with the optimal processing time under various system parameters, such as number of machines, speed of machines and the associated overheads, are presented in figure 4. SIP is a simplistic model and it is easy to realize in practice. Equation (15) determines the maximum number of machines that can be used, given a material quantity  $L$  to be processed. Thus, this serves as a *necessary and sufficient* condition for utilizing all the  $N$  machines to process the entire material. Thus, when the condition in (15) fails to hold, then we determine whether or not the remaining  $N - 1$  machines can be utilized. Thus, with at most  $O(N)$ , we obtain the maximal set of machines that can be utilized to process the entire material in an optimal time. Figure 4 shows the impact of  $C$  with a heterogeneous set of machines. Thus, depending on the parameters, the optimal number of machines that can be used is identified. As expected, for smaller  $C$  values, the maximal number of machines that can be utilized is fairly large. Figure 4(b) shows the behaviour of the overhead with respect

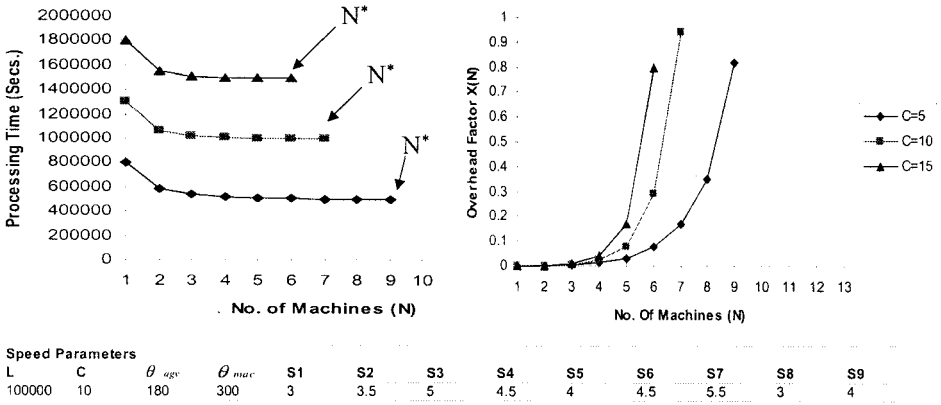


Figure 4. Behaviour of processing time and overhead factor in a heterogeneous system with  $N$  machines.

to the number of machines for various  $C$  values. From figure 4(a) we observe that for the system with larger  $C$  values,  $N^*$  is smaller. This is due to the fact that larger value of  $C$  affects the overhead factor more adversely, as shown in figure 4(b). For instance, from figure 4(a), it may be noted that when the  $C$  value decreases from 10 to 5,  $N^*$  increases by a significant amount and allows us to utilize two more machines to gain approximately a 39% decrease in completion time. We can also observe a similar trend in overhead reduction from figure 4(b).

For homogeneous systems, we have derived several interesting results. One of the useful relationships is in determining the choice of a particular AGV used for distributing the material. Thus, if the transit time delay of a given AGV (it may be the worst case delay or the average delay) is more than (21), then utilizing this particular AGV to distribute to all the  $N$  machines will incur more overheads and will result in a non-optimal performance.

When a sequence of material distribution is fixed among the machines, we tend to eliminate the processors starting from  $N$  until condition (15) is satisfied. However, when we are allowed to choose any sequence of material distribution, one can utilize the *greedy algorithm* described. Thus, sequencing of material delivery to MCs also makes an impact on the overall processing time. From (16), we observe the influence of the sequence in which the material is delivered to MCs. The greedy algorithm proposes a way to choose a sequence that produces an optimal processing time. It states that the distribution that follows the order in which the processing speeds of the machines decrease, will result in an optimal processing time. Of course, condition (15) has to be verified to utilize the existence of an optimal processing time employing  $N$  machines. It should be noted that optimization by sequencing of material delivery is a generic conclusion. It may not apply all in cases, especially when the overheads are drastically different at different MCs. In such cases, it is possible that optimal processing time is achieved by utilizing fewer than all the available machines. Example 2 highlights this important aspect. It demonstrates the fact that a subset of machines may complete the processing in a time less than when using large number of machines using an arbitrary sequence, when a fast sequence fails to exist.

The main problem with SIP is that it causes high idle time at the MCs, as they have to wait until the raw material is delivered to them. MIP minimizes this idle time

at MCs. This reduction in idle time is done by delivering a portion of the material to MCs early so that they can start processing early. The entire material for a MC in this policy is delivered in several instalments. Another importance of MIP is in its applicability to real-life scenarios, wherein, AGVs have limited material carrying capacity. Thus, for a given amount of material, if the maximum amount of load assigned to any machine in any particular instalment is less than the maximum capacity of the AGV, then MIP is a better strategy to adopt. Some extensions to the problem addressed in this paper are mentioned in the next section.

## 6. Conclusions

This paper extends the research currently being done in the field of scheduling and routing of AGVs in FMS. The ultimate aim is to achieve higher manufacturing efficiencies thus driving product costs down. The significant contributions of this paper are as follows. We introduced a Directed Flow Graph representation to describe the material distribution process. The flow graph representation is a simple and elegant graphical means to describe the features of the load distribution strategy. We built a mathematical model to obtain the optimal material distribution. We considered various factors, such as the heterogeneous processing speed of machines, the overheads, such as the start-up delay, transportation delay, and loading/unloading delay. We designed a single-instalment policy (SIP) and derived a closed-form solution for the optimal processing time. We derived a necessary and sufficient condition for the existence of an optimal processing time with a given system of  $N$  machines. Further, we considered the effect of sequencing on the processing time. The analysis presented for SIP has been extended for the case of a multi-instalment policy (MIP), wherein the material is transported to the MCs in more than one instalment. This is shown to minimize the overall processing time of the entire material further.

Several extensions of this work seem to be plausible. As a first extension, it would be interesting to consider the capacity of the AGV explicitly in the problem formulation, particularly in the design of MIP. The solution to this problem will then be practically useful in a real-life situation. Of course, the proposed strategy would work comfortably if we assume that the capacity of the AGV is large enough to accommodate the largest portion of the material distribution. Another extension to the proposed SIP and MIP could be to design a Hybrid Instalment Policy (HIP), where all the available machines in the FMS are grouped in a few clusters. This clustering could be on the basis of some common characteristics of the machines in a cluster or could simply be a dependency of the work in progress to facilitate routing. The AGV is then constrained to deliver to all the machines in a cluster, either using SIP or MIP. Further research could also focus on modelling several heterogeneous AGVs in the system. These AGVs could have different speeds on the shop floor and different material carrying capacities. Such factors would pose additional constraints in formulating the optimal material distribution.

## References

- KAIGHOBADI, M. and VENKATEST, K., 1994, Flexible manufacturing systems: an overview. *International Journal of Operation and Production Management*, **14**(4), 25–49.
- VISWANADHAM, N. and NARAHARI, Y., 1998, *Performance Modeling of Automated Manufacturing Systems* (Prentice Hall of India), pp. 11–151.

- SEO, Y. and EGBELU, P. J. 1999, Integrated manufacturing planning for an AGV based FMS. *International Journal of Production Economics*, **60–61**, 473–478.
- SURI, R. and DESIRAJU, R., 1997, Performance analysis of material handling systems with a single discrete material handling device. *The International Journal of Flexible Manufacturing Systems*, **9**, 223–227.
- REVELIOTIS, S. A., 2000, Conflict resolution in AGV systems. *IIE Transactions*, **32**, 642–657.
- SOYLU, M., OZDEMIREL, N. E. and KAYALIGIL, S., 2000, A self-organizing neural network approach for the single AGV routing problem. *European Journal of Operational Research*, **121**, 124–137.
- TSUBONE, H. and HORIKAWA, M., 1999, A comparison between machine flexibility & routing flexibility. *The International Journal of Flexible Manufacturing Systems*, **11**, 83–101.
- CAPRIHAN, R. and WADHWA, S., 1997, Impact of routing flexibility on the performance of an FMA—a simulation study. *The International Journal of Flexible Manufacturing Systems*, **9**, 273–298.
- KATS, V. and LEVNER, E., 1998, Cyclic scheduling of operations for a part type in a FMS handled by a single robot: a parametric critical path approach. *The International Journal of Flexible Manufacturing Systems*, **10**, 129–138.
- MAHMOODI, F., MOSIER, C. T. and MORGAN, J. R., 1999, The effects of scheduling rules and routing flexibility on the performance of a random flexible manufacturing system. *The International Journal of Flexible Manufacturing Systems*, **11**, 271–289.
- PIRAMUTHU, S., SHAW, M. and FULKERSON, B., 2000, Information based dynamic manufacturing system scheduling. *The International Journal of Flexible Manufacturing Systems*, **12**, 219–234.
- KIM, C. W., TANCHOCO, J. M. A. and KOO, P. H., 1999, AGV dispatching based on workload balancing. *International Journal of Production Research*, **37(17)**, 4053–4066.
- HOFF, E. B. and SARKER, B. R., 1998, An overview of path design and dispatching methods for automated guided vehicles. *Integrated Manufacturing Systems*, **9(5)**, 296–307.
- CHEN, F. F., HUANG, J. and CENTENO, M. A., 1999, Intelligent scheduling and control of rail guided vehicles and load/unload operations in a flexible manufacturing system. *Journal of Intelligent Manufacturing*, **10**, 405–421.
- SAYGIN, C. and KILIC, S. E., 1999, Integrated flexible process plans with scheduling in flexible manufacturing systems. *The International Journal of Advanced Manufacturing Technology*, **15**, 268–280.
- MOON, S. W. and HWANG, H., 1999, Determination of unit load sizes of AGV in multi-product, multi-line assemble production systems. *International Journal of Production Research*, **37(15)**, 3565–3581.
- RAJOTIA, S., SHANKER, K. and BATRA, J. L., 1998, Determination of optimal AGV fleet size for FMS. *International Journal of Production Research*, **36(5)**, 1177–1198.
- CHAN, F. T. S., 1999, Evaluations of operational control rules in scheduling a flexible manufacturing system. *Robotics and Computer Integrated Manufacturing*, **15**, 121–132.
- OBOOTH, C., BATTI, R. and KARWAN, M., 1999, Dynamic conflict free routing of automated guided vehicles. *The International Journal of Production Research*, **37(9)**, 2003–2030.
- BUKCHIN, J. and TZUR, M., 2000, Design of flexible assembly line to minimize equipment cost. *IIE Transactions*, **32**, 585–598.