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Achieving sharp deliveries in supply chains through variance pool allocation

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Abstract

Variability reduction and business process synchronization are acknowledged as key to achieving sharp and timely deliveries in supply chain networks. In this paper, we develop an approach that facilitates variability reduction and business process synchronization for supply chains in a cost effective way. The approach developed is founded on an analogy between mechanical design tolerancing and supply chain lead time compression. We first present a motivating example to describe this analogy. Next, we define, using process capability indices, a new index of delivery performance called *delivery sharpness* which, when used with the classical performance index *delivery probability*, measures the accuracy as well as the precision with which products are delivered to the customers. Following this, we solve the following specific problem: how do we compute the allowable variability in lead time for individual stages of the supply chain so that specified levels of delivery sharpness and delivery probability are achieved in a cost-effective way? We call this the variance pool allocation (VPA) problem. We suggest an efficient heuristic approach for solving the VPA problem and also show that a variety of important supply chain design problems can be posed as instances of the VPA problem. One such problem, which is addressed in this paper, is the supply chain partner selection problem. We formulate and solve the VPA problem for a plastics industry supply chain and demonstrate how the solution can be used to choose the best mix of supply chain partners.
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Keywords: Supply chain management; Lead time reduction; Variability reduction; Process capability indices; Statistical tolerancing; Variance pool allocation (VPA)

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1. Introduction

Businesses today operate in a very tough environment that is constantly in flux [1,2]. Customers have become increasingly demanding looking for better and innovative goods and services that are specifically customized to meet their unique needs. There is also an implicit requirement on the accuracy, timeliness, convenience, responsiveness, quality and reliability of the service offered to them. And all of this is desired at ever-lower prices. Simultaneously, the rapid pace of innovation has resulted in shorter product and technology cycles, leading to uncertainties in supply and demand. Variability is thus a major issue and variability reduction and business process synchronization are therefore acknowledged as key to achieving superior levels of performance in supply chain networks. This paper proposes an approach inspired by statistical design tolerancing for achieving cycle time compression in supply chains through variability reduction.

One of the key issues in supply chain design, facing companies today is the strategic selection of partners for each stage of their outsourced value chain, in the face of uncertainties of various kinds [3,4]. This selection needs to take into account the synchronization of schedules for suppliers, manufacturers, and logistics providers in order to streamline processes throughout the supply chain. The variability reduction approach presented in this paper focuses on this important problem in supply chain design.

1.1. Contributions

The main contribution of this paper is in suggesting a way of formulating design optimization problems in supply chains by exploiting the connections with statistical design tolerancing. This opens up the use of statistical design tolerancing techniques and tools to be used in supply chain design and optimization. The specific contributions of this paper can be summarized as follows:

1. We first present a motivating example to develop an analogy between mechanical assemblies and supply chain networks. The example shows that the variation in end-to-end lead time of a supply chain can be viewed as the variation in the dimension of the parts produced by a machining process.

2. The above example motivates us to investigate the use of standard design tolerancing techniques (based on process capability indices), that are popularly used for quantifying and reducing the defective assemblies produced by a machining process, for the purpose of quantifying the delivery performance of the supply chain. Using supply chain process capability indices, we describe the delivery performance of a supply chain in terms of two metrics. The first is a traditional metric, *delivery probability* (DP), which is the probability that a typical customer order is delivered during a customer-specified window. The second metric is a new one that we propose, which we refer to as *delivery sharpness* (DS), which is a measure of how close to the target (most desired) delivery date a customer order is actually delivered.

3. The setup above prepares the ground for formulating the following generic design optimization problem for supply chains:

> Given a supply chain and the mean and standard deviation of the end-to-end lead time for a certain product mix, how do we optimally distribute the pool of variance among individual business processes so as to minimize the cost and achieve six sigma delivery performance?

We call this problem as the variance pool allocation (VPA) problem. We come up with a five stage approach for solving the VPA problem. We then look at linear or pipelined supply chains and solve the VPA problem through the Lagrange multiplier method.

4. Finally, we show that a rich variety of supply chain design problems, in particular, the supply chain partner selection problem, can be cast as a VPA problem. We show that the optimal variance of each stage

obtained by solving the VPA problem can be used in selecting the best mix of partners out of a possible set of alternatives for each stage of the supply chain. To substantiate this, we consider a six stage supply chain in the plastics industry, for which we formulate and solve the VPA problem. We show how a manager can use the solution of the VPA problem, in order to select the best combination of supply chain partners: supplier, manufacturer, inbound logistics provider, assembler and outbound logistics provider out of a given set of choices, so as to ensure timely delivery of finished products to customer destinations.

1.2. Related work

The subject matter of this paper falls in the intersection of following areas of current interest: (1) variability reduction and lead time compression techniques for business processes, (2) statistical design tolerancing, and in particular, theory of process capability indices, (3) the Motorola six sigma program, and (4) Taguchi methods.

Lead time compression in business processes is the subject matter of a large number of papers in the last decade. See for example, the papers by Hopp et al. [5]; Adler et al. [6]; Narahari et al. [7]; and Chao and Graves [8]. Variability reduction is a key strategy used in the above papers and other related papers. Hopp and Spearman, in their book [9], have brought out this key role played by variability reduction. Lead time compression in supply chains is the subject of several recent papers, see for example, Narahari et al. [10]; Garg et al. [11].

Statistical design tolerancing is a mature subject in the design community. The key ideas in statistical design tolerancing which provide the core inputs to this paper are: (1) theory of process capability indices [12–15]; (2) tolerance analysis and tolerance synthesis techniques [16–18]; (3) Motorola six sigma program [19,20]; (4) Taguchi methods [21,22]; and (5) design for tolerancing [7,23,24].

Variability reduction in supply chains, especially in the context of inventory optimization and delivery performance, is the topic of several papers in the past decade. Important ones of relevance here are [25–36]. The article by Schwartz and Weng [28] is particularly relevant here. This paper discusses the joint effect of lead time variability and demand uncertainty, as well as the effect of "fair-shares" allocation, on safety stocks in a four-link JIT supply chain. Masters [32] developed in 1993 an optimization model to determine near optimal stock levels for multi-echelon distribution inventories. His formulation also uses variability reduction principles. Ettl et al. [33] develop an inventory-queue model of a multi-echelon supply chain with base stock policy followed at each store. Given the bill of materials, the nominal lead times, the demand data, and the cost data, their model generates the base stock level at each store that minimizes the overall inventory capital in the network and guarantees the customer service requirements. Chopra et al. [31] study the effect of lead time uncertainty on safety stocks in a multi-echelon supply chain. The articles by Song and co-authors [25–27,29] essentially analyze the effect of stochastic lead times on the performance of assemble-to-order systems. The volume edited by Tayur et al. [34] also contains several inventory optimization models in the supply chain context, where variability reduction is discussed as a key to improving the supply chain delivery performance.

The salient feature of our work which distinguishes it from all the above discussed models, is the notion of six sigma quality for the end-to-end delivery process. Existing models in the literature consider either the availability of product to the customer as a criterion for customer service level or probability of delivering the product to the customer within a window as a measure of customer's service level. Away from these classical measurements of customer service levels in the inventory optimization problem, we propose an approach for customer service level, namely accuracy and precision of deliveries, which is the primary objective of any modern supply chain. Also, the present paper uses key ideas and notions in the area of statistical design tolerancing in achieving variability reduction and synchronization in the supply chain process, leading to quicker and sharper deliveries.

A preliminary version of this paper [11] contains some but not all of the ideas presented in this current paper. A companion paper [37] explores these ideas in a different direction and applies the ideas to inventory optimization in multi-stage supply chains.

2. An analogy between mechanical assemblies and supply chain networks

A typical value delivery process in a supply chain starts with an order from the customer and ends with customer satisfaction. This process consists of a series of activities, each performed by various subsystems. There is an analogy between the structure of a complex mechanical assembly and the structure of the supply chain process. A supply chain is like a complex mechanical assembly; it is a conglomeration of numerous business processes just like a complex assembly is an arrangement of numerous subassemblies. Figs. 1 and 2 show a serial and converging–diverging supply chain respectively together with analogous mechanical assemblies. The analogy arises from the fact that we are interested in analyzing end-to-end delivery performance of the supply chain. Among the performance measures of a business process, end-to-end lead time is perhaps the most important and is the major theme of this paper. The lead time performance of

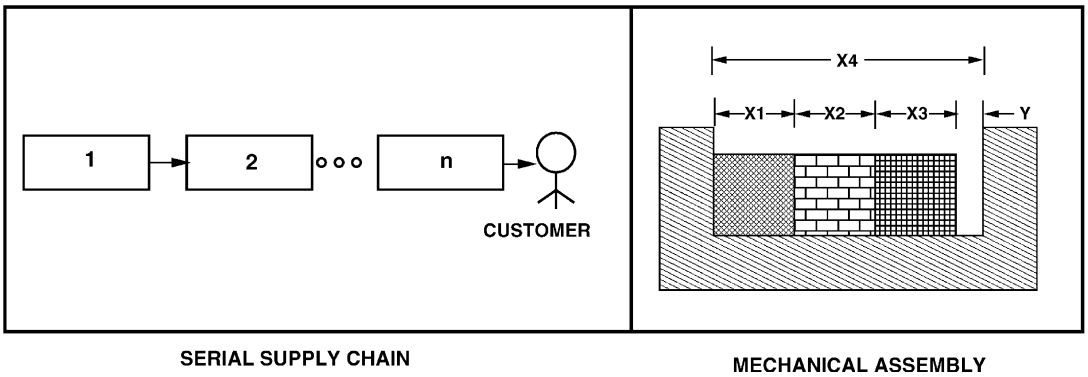

Fig. 1. An analogous mechanical assembly for a serial supply chain.

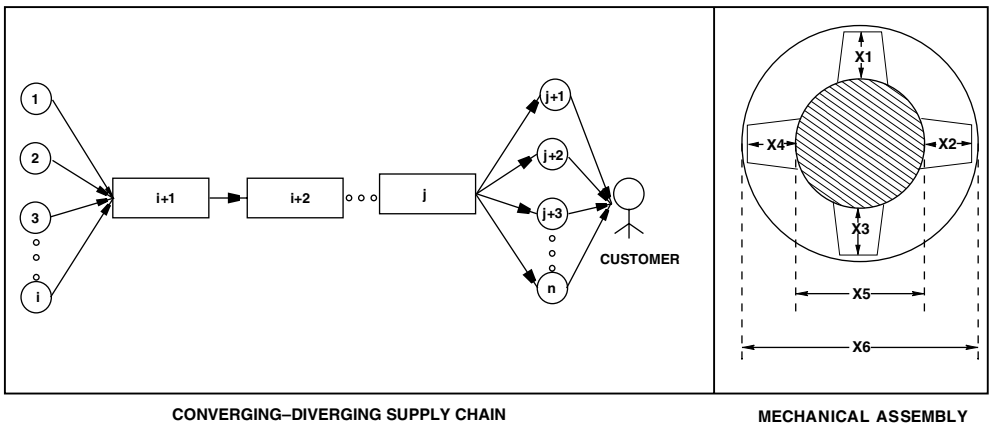

Fig. 2. An analogous mechanical assembly for a converging–diverging supply chain.

any business process depends not only on how long it takes to provide a service to the customer but also how much it varies from one customer to another. Lead time and its variation for individual work processes are key determinants of end-to-end delivery performance of a supply chain network. When the number of resources, operations, and organizations in a supply chain increases, variability destroys synchronization among the individual processes, leading to poor delivery performance. On the other hand, by reducing variability all along the supply chain in an intelligent way, proper synchronization can be achieved among the constituent processes.

To illustrate the above fact, let us consider a linear supply chain as shown in Fig. 1 where material flows through n different processes before it is delivered as finished product to the end customer. Assuming that no inventory is maintained at any intermediate stage of the supply chain, the end-to-end lead time, say Y, of an end customer order becomes equal to the sum of processing times (lead time) of individual processes, say X_i's. That is

$$Y = \sum_{i=1}^{n} X_i.$$

Note that depending upon requirement one can decompose each business process into a hierarchy of low level processes. In that situation, the supply chain will look like a complex network of business processes and the end-to-end lead time Y will depend on individual process lead time in a more complex way. An instance of such complex supply chain network is shown in Fig. 2 where we are considering a simple converging–diverging supply chain network. For this network, the end-to-end lead time Y of a customer order is given by following expression (assuming no inventory is maintained at any of the stages):

$$Y = \max(X_1, \ldots, X_i) + \sum_{p=i+1}^{j} X_p + \min(X_{j+1}, \ldots, X_n).$$

Note that if other factors such as inventory replenishment policy, demand uncertainty, etc. are also taken into account then Y will become an even more complex function of the system parameters.

To understand the analogy between structure of mechanical assembly and structure of supply chain network, let us consider a mechanical assembly shown in Fig. 1 where the objective is to control the gap (target dimension) Y. This dimension is dependent on the dimension of the other parts as well as the configuration of the assembly in the same way as the end-to-end lead time Y was dependent on the lead time of individual business processes and other system parameters. It is easy to see that the gap Y can be expressed in terms of dimensions of its subassemblies in the following manner:

$$Y = X_4 - \sum_{i=1}^{3} X_i.$$

The objective for the mechanical assembly presented in Fig. 2 is also to control the gap, denoted by Y, between the circular casing and the blades of the fan. Like in the previous case, this dimension is dependent on the dimension of the other parts as well as the configuration of the assembly. The configuration of this assembly is more complex than previous one and the target dimension Y is dependent on the parts dimensions in the following way:

$$Y = X_6 - X_5 + \max(X_1, \ldots, X_4).$$

It is a well known fact that during the machining process, the dimensions of individual parts keep varying from parts to parts which ultimately results in variation in target dimension Y of the assembly. The same is the case with supply chain lead time. In the context of supply chain networks, the lead times of individual business processes are variable in nature and so is Y. The control on variability in Y is the key to achieving outstanding delivery performance.

In this paper, we use the above analogy between mechanical assemblies and supply chain processes for measuring and controlling the lead time variations in the supply chain. For this we use the techniques of statistical design tolerancing. In particular, we use the notion of process capability indices C_p, C_{pk} and C_{pm}, the notion of Motorola six sigma quality, Taguchi methods, and other best practices in design tolerancing. In the next section, we set up this framework in the supply chain context.

3. A process capability perspective for supply chain performance

The process capability indices C_p, C_{pk}, and C_{pm} [12] are popular in the areas of design tolerancing and statistical process control. Let us consider the situation depicted by Fig. 3 in order to describe the idea of how capability of a process, where variability is an inherent effect, can be measured. Below is the list of symbols used in this figure.

X	lead time or any general quality characteristic X
μ	mean of X
σ	standard deviation of X
L	lower specification limit of customer delivery window
U	upper specification limit of customer delivery window

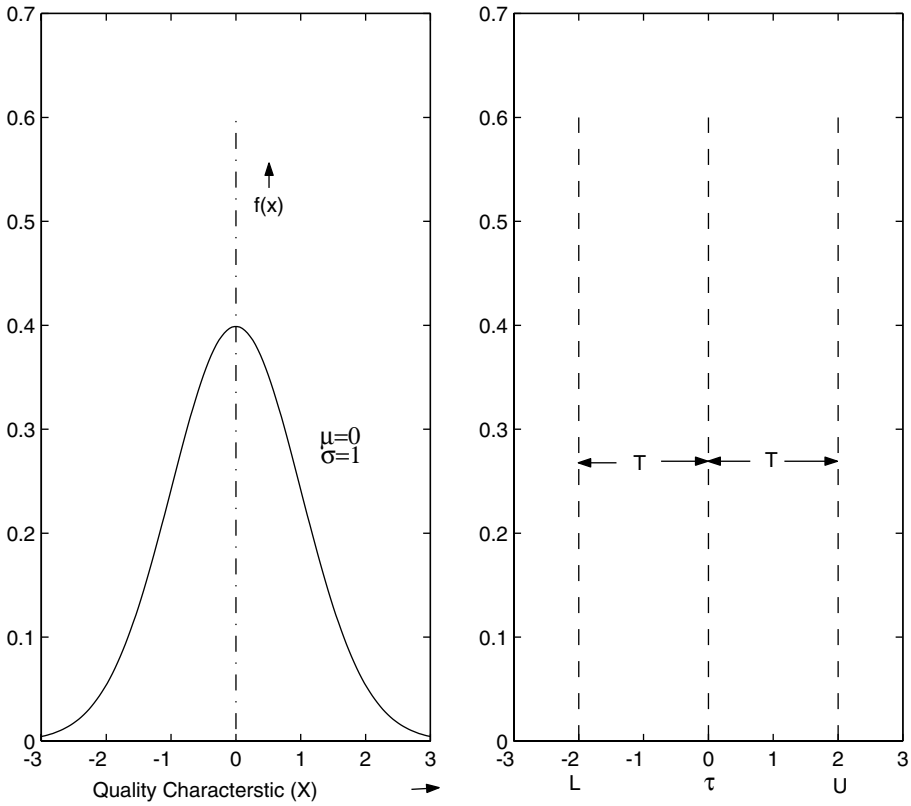

Fig. 3. Process variability and customer delivery window.

τ	target value for X, specified by customer
T	tolerance for X, specified by customer
b	bias $= \lvert\tau - \mu\rvert$
d	$\min(\lvert U - \mu\rvert, \lvert\mu - L\rvert)$

In this figure, variability of the process is characterized by the probability density of the quality characteristic X produced by the process, and customer specifications are characterized by a delivery window which consists of tolerance T and target value τ. Normal distribution is a popular and common choice for X because of its fundamental role in the theory of process capability indices. The target value τ can be any value between L and U but we have assumed it as the mid point of two limits for the sake of convenience, that is $\tau = \frac{U+L}{2}$.

In what follows, we present relevant definitions and present a quick summary of relevant results for the process capability indices C_p, C_{pk}, and C_{pm}. See [38,14,37,15,39] for more details.

3.1. Supply chain process capability indices

The three indices C_p, C_{pk}, and C_{pm} are defined in following way:

$$C_p = \frac{U - L}{6\sigma} = \frac{T}{3\sigma}, \tag{1}$$

$$C_{pk} = \frac{\min(U - \mu, \mu - L)}{3\sigma} = \left(\frac{d}{3\sigma}\right), \tag{2}$$

$$C_{pm} = \frac{U - L}{6\xi} = \frac{T}{3\sqrt{\sigma^2 + b^2}}. \tag{3}$$

The following relations can easily be derived [39] among all the three indices: C_p, C_{pk}, and C_{pm}. See Appendix A for proof of (5) and (6).

$$C_p \geqslant C_{pk} \geqslant 0; \quad C_p \geqslant C_{pm} \geqslant 0, \tag{4}$$

$$C_{pk} = C_p(1 - k), \quad \text{where } k = \frac{b}{T}, \tag{5}$$

$$\frac{1}{9C_{pm}^2} = \frac{1}{9C_p^2} + \left(1 - \frac{C_{pk}}{C_p}\right)^2. \tag{6}$$

Two important quantities: *potential* and *actual yield* of a process will play a critical role in the development of optimization problems. We, define these quantities below.

Actual yield: The probability of delivering a product within a specified interval.

Potential: The probability of delivering a product within a specified interval, if the process distribution is centered at the target value, i.e. $\mu = \tau$.

It is easy to prove [39] the following relations (see Appendix A for their proof):

$$\text{Potential} = 2\Phi(3C_p) - 1, \tag{7}$$

$$\text{Actual yield} = \Phi(3C_{pk}) + \Phi(6C_p - 3C_{pk}) - 1, \tag{8}$$

where $\Phi(\cdot)$ is the cumulative distribution function of standard normal distribution.

3.2. Delivery probability and delivery sharpness

It can be verified that a unique (C_p, C_{pk}) pair results in a unique actual yield, therefore, the 3-tuple (C_p, C_{pk}, C_{pm}) can be substituted by the pair (Actual yield, C_{pm}) to measure the delivery quality. Being an indicator for precision and accuracy of the deliveries, we prefer to call actual yield of the process as *delivery probability* (DP) and C_{pm} as *delivery sharpness* (DS). In the present paper, we use these two indices to measure the quality of any delivery process in a given supply chain.

Motivated by the Motorola six sigma (MSS) program, we prefer to express DP in terms of $\theta\sigma$ levels, where $\theta \in \Re^+$, rather than expressing it in terms of numerical values. In the MSS program, each $\theta\sigma$ level corresponds to a unique number in the interval $[0, 1]$ and these numbers actually corresponds to upper bounds on the yield of the process. However, here we are assuming that these numbers correspond to the actual yield of the process. For example, according to the MSS program, in the presence of process mean shifts and drifts, if upper bound on yield of the process is equal to $1–3.4 \times 10^{-6}$ then its quality is 6σ quality. In the framework of DP and DS, the DP of a process is 6σ iff its actual yield is $1–3.4 \times 10^{-6}$. Moreover, in the framework of DP and DS no shifts and drifts are allowed in process mean, only bias is allowed between process mean and target value. It is easy to see that the upper bound in the MSS program for $\theta\sigma$ level is $\Phi(\theta-1.5)$. Equating this to the actual yield of the process we get the following equation for $\theta\sigma$ quality curve on the C_p–C_{pk} plane. Some of these curves are plotted in Fig. 4.

$$\Phi(\theta - 1.5) = \Phi(3C_{pk}) + \Phi(6C_p - 3C_{pk}) - 1.$$

We can investigate the connection between delivery probability and delivery sharpness in the following way. Consider the plots of σ quality levels on C_{pk}–C_p plane and then see how C_{pm} behaves on the same plot. For this purpose we use the identity relation (6) among C_p, C_{pk}, and C_{pm} and plot this relation for a constant value of C_{pm} (say C^*_{pm}). The plot comes out to be a section of a hyperbola. From a process design point of view, it can be said that for a desired minimum level of DS (i.e. C_{pm}) and DP (i.e. C_p, C_{pk}), this curve provides a set of 3-tuples (C_p, C_{pk}, C_{pm}) which all satisfy these two requirements. The designer has to decide which one of the triples to choose depending upon the requirements. Fig. 4 shows some C_{pm} curves on the C_p–C_{pk} plane.

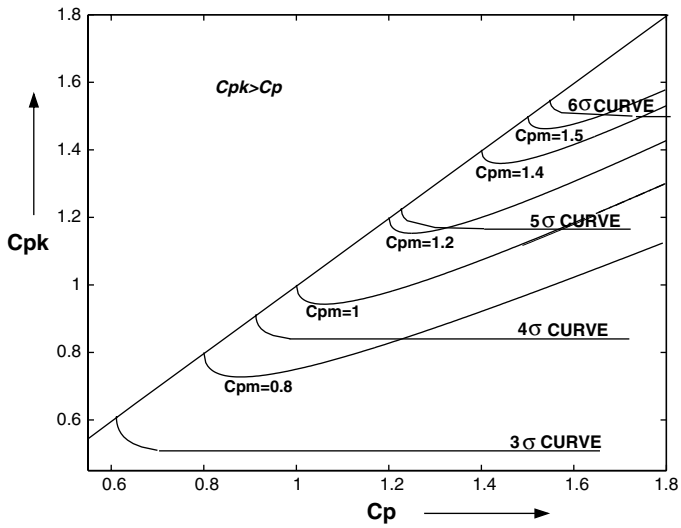

Fig. 4. $\theta\sigma$ curves and C_{pm} curves on C_p–C_{pk} plane.

4. Variance pool allocation problem

Using the paraphernalia developed in the previous section, we can formulate mathematical programming problems to describe design optimization and tactical decision making in supply chain networks. We now set out to define a particular design optimization problem, which we refer to as the variance pool allocation problem. For the sake of simplicity, we consider make-to-order supply chains where the flow of materials is linear. The methodology can be easily extended (with extra computational requirements) to more general supply chains.

4.1. Description of the problem

Consider a linear, make-to-order (MTO) supply chain with n stages as shown in Fig. 5. This supply chain is a single product supply chain. The product is delivered to the end customer from stage n. In the present model we are not concerned about how the orders are consolidated, how the production planning is done, and at which intermediate stages finished goods or semi-finished goods inventory will be maintained. Let us assume that as soon as any customer places an order for a unit of the product, the flow of material against the order starts from stage 1, undergoes processing at successive stages, and is finally delivered to the customer after processing at stage n. Let the lead time at each stage be considered as a continuous random variable (i.e. X_i, $i = 1, 2, \ldots, n$). As a consequence of this assumption, the end-to-end lead time is also a continuous random variable. Note that, in Fig. 5, we have shown the probability density function of random variable X_i just above the corresponding process i ($i = 1, 2, \ldots, n$).

The first objective of the study here is to find out how the lead time variance of individual stages should be chosen, assuming that the mean lead time is given for each stage, such that the specified levels of DP and DS are attained for a given end-to-end lead time delivery window in a cost effective way. This is referred to as the variance pool allocation (VPA) problem.

Depending on the nature of the objective function chosen, the solution of the VPA problem can be used in a wide variety of tactical decision making in supply chains. Typical such problems include: due date setting, choice of customers, inventory allocation, vendor selection, choice of logistics modes, choice of logistics providers, and choice of manufacturing control policies. The second objective of our study here is to explain one such compelling application of the VPA problem. As an example, we show how the solution of the VPA can be used in choosing the best possible mix of alternatives for supply chain operation. This is referred to as the supply chain partner selection problem. In what follows is a crisp idea behind how one can use the solution of the VPA problem for solving the supply chain partner selection problem.

Let us assume that each stage of the supply chain is a work process, e.g. transportation, machining, procurement, etc., then it is realistic to assume that in general there are number of service providers available

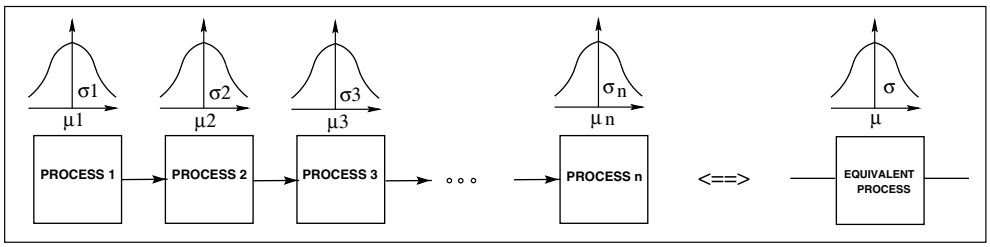

Fig. 5. A linear supply chain model.

for each individual stage. For example, there might be several logistics providers in the fray and each one of them can ship goods from one place to another. Typically, mean and variance of the shipment time vary from one candidate service provider to another and so is the freight charges. As a special case, assume that all candidate service providers promise for the same mean shipment time but quote the different variabilities. These variances may be available through past history also. If we thus know the pairs of cost and variance for each candidate service provider, the pairs can be used to fit a polynomial cost function in terms of the variance of shipping time. Such functions can be obtained for each stage of the supply chain. Knowing all these cost functions, the minimization of total cost of the supply chain will result in optimal values of variance for each stage. These optimal values can be used to pick up at most two service providers for each stage whose variance is closest to the corresponding optimal value. Thus the size of the problem of selecting optimal mix of service providers along the supply chain can be reduced to a case where we need to consider at most two candidates per stage. The best mix can now be easily obtained (for example by exhaustive enumeration). The best mix obtained will be optimal in the sense that it results in desired levels of DP and DS on the end-to-end delivery process with minimum possible total coat.

4.2. Assumptions

The model for VPA, proposed in this paper, is based upon the following assumptions about the nature of the business process and the customer delivery window:

1. Lead time X_i at stage i ($i = 1, \ldots, n$) is normally distributed, say, with mean μ_i and standard deviation σ_i. This assumption is quite standard and has received widespread justification in the literature through practical experimentation (see for example [20]). This assumption also has a theoretical basis through the central limit theorem.
2. Lead times X_i are mutually independent. This assumption can be justified on two grounds: (a) The individual lead times correspond to essentially entire subsystems or subprocesses in the supply chain network. Our view of the supply chain network is at a high level of abstraction and the lead time of a typical subsystem is determined completely by the dynamics internal to that subsystem once the inputs arriving from the previous stage are given. (b) The supply chain network can be aptly modeled as a tandem queueing network. From the theory of product form queueing networks (in particular Jackson networks) [40], it is known that, under stable conditions, the lead times of individual stages of a product form network behave as if they are independent.
3. There is no time elapsed between end of process i to commencement of process $i + 1$ ($i = 1, \ldots, n - 1$). This is a reasonable assumption since any interface time can be absorbed into the lead time of stage i or lead time of stage $i + 1$. As an immediate consequence of this assumption, the end-to-end lead time, Y, is equal to the sum of lead times of the individual processes:

$$Y = \sum_{i=1}^{n} X_i. \tag{9}$$

   Y can be easily seen to be normally distributed with $\mu = \sum_{i=1}^{n} \mu_i$ and $\sigma^2 = \sum_{i=1}^{n} \sigma_i^2$ since it is the sum of n independent normally distributed random variables.
4. Each customer specifies a customer delivery window (τ, T) where τ is the desirable amount of the time a customer is willing to wait after placing the order. The customer is prepared to wait for a maximum period of $\tau + T$. Also, the customer does not want the delivery to occur before $\tau - T$. Therefore, τ is the target value for end-to-end delivery process and T is the tolerance.

4.3. Formulation of the VPA problem

Essentially the VPA problem is a mathematical programming problem, hence it can be defined very well in the form of known parameters, decision variables, objective function, and constraints.

4.3.1. Known parameters

The following parameters are known in a typical VPA problem.

1. End customer delivery window (τ, T).
2. Mean μ_i of random variable X_i, $i = 1, 2, \ldots, n$.
3. Delivery probability and delivery sharpness for end-to-end lead time, Y.
4. Processing cost per unit product at each stage i, denoted by $\mathscr{K}_i$, of the supply chain. This cost is the part of the total processing cost that is associated with lead time. For example, in the case of a manufacturing process, it could be the opportunity cost of capital tied up with machinery. Similarly if it is the logistics process, it may represent the cost of transportation itself. As we showed earlier that for a given stage i, the mean processing time μ_i is almost same for all the potential service providers but their variance may differ and hence per unit processing cost may also vary. Assume that for each stage i, the processing time variances $\sigma_{iA}, \sigma_{iB}, \ldots$ and per unit processing costs $C_{iA}, C_{iB}, \ldots$ are known for all the service providers $A, B, \ldots$ of that stage. The pairs $(\sigma_{iA}, C_{iA}), (\sigma_{iB}, C_{iB}), \ldots$ can be used to get a polynomial function for per unit processing cost in terms of σ_i. For the sake of conceptual and computational simplicity we are motivated to choose a second order polynomial in the following manner:

$$\mathscr{K}_i = A_{i0} + A_{i1}\sigma_i + A_{i2}\sigma_i^2. \tag{10}$$

   Here A_{i0}, A_{i1}, A_{i2} are constants and not all are positive. These constants will be obtained by polynomial curve fitting for the pairs $(\sigma_{iA}, C_{iA}), (\sigma_{iB}, C_{iB}), \ldots$. Thus, at an abstract level we can safely assume that these constants are given to us for each stage of the supply chain.

Here we would like to make an important remark. In most of the practical cases, getting the values for the constants A_{i0}, A_{i1}, A_{i2} is not an easy task. First of all, the shipment time of a given service provider may not be normally distributed and moreover, neither the shipper nor the carrier may be keeping a record of past data of shipment times.

4.3.2. Decision variables

The decision variables of the VPA problem are optimal standard deviations σ_i^* of each individual stage i $(i = 1, \ldots, n)$. As we mentioned the scheme earlier, these optimal standard deviations σ_i^* can be used to compute the optimal partners (or service provider) $P_1^*, P_2^*, \ldots, P_n^*$ for each stage of the supply chain.

4.3.3. Objective function and constraints

As stated already, the objective in the VPA problem is to minimize the cost and the constraints are specified in terms of minimum expected levels of DP and DS on end-to-end lead time.

In the present model, we have confined our discussion only to lead time variability of the supply chain without considering other issues like demand variability, inventory levels, etc. Therefore, it seems to be reasonable to consider the total processing cost of a single unit of product, denoted by $\mathscr{K}$, as the objective function. This cost is simply the sum of processing costs of all the stages. Thus the problem formulation becomes:

Minimize

$$\mathscr{K} = \sum_{i=1}^{n} \mathscr{K}_i = \sum_{i=1}^{n} (A_{i0} + A_{i1}\sigma_i + A_{i2}\sigma_i^2) \tag{11}$$

subject to

$$\text{DS for end-to-end lead time} \geqslant C_{pm}^*, \tag{12}$$

$$\text{DP for end-to-end lead time} \geqslant 6\sigma, \tag{13}$$

$$\sigma_i > 0 \quad \forall i. \tag{14}$$

4.4. A 5-step solution approach

In this section, we present a 5-step procedure for solving any specific design optimization problem. the first four steps constitute the solution of the VPA problem and the fifth step explains how this solution can be used to solve the specific problem at hand. In this case, the specific problem we discuss is the supply chain partner selection problem.

4.4.1. Step 1: Problem formulation

The first step in solving the VPA problem is to collect all the known parameters specified in the problem and then formulate the problem in terms of a non-linear optimization problem as presented in Section 4.3.3. This includes:

- Extracting the value of μ_i, τ, and T.
- Extracting the desired level of DP and DS for end to end lead time.
- Obtaining the pairs $(\sigma_{iA}, C_{iA}), (\sigma_{iB}, C_{iB}), \ldots$ for each stage i of the chain and then fitting it to get second order polynomial $\mathscr{K}_i$.

4.4.2. Step 2: Expressing the constraints in terms of decision variables

Note that the first two constraints in the optimization problem, formulated in Step 1, are not being expressed in terms of decision variables. This step does the job of expressing the constraints in terms of decision variables.

Recall the following expression from Section 4.2:

$$\sigma^2 = \sigma_1^2 + \sigma_2^2 + \cdots + \sigma_n^2, \tag{15}$$

where σ is the standard deviation for end-to-end lead time. This can be expressed in terms of C_p^* and C_{pk}^* of end-to-end lead time Y in following manner:

$$\sigma^2 = \frac{T^2}{9C_p^{*2}} = \frac{d^2}{9C_{pk}^{*2}}, \tag{16}$$

where T, the tolerance of end customer delivery window, is a known parameter, and d, given by $\min(\tau + T - \mu, \mu - \tau + T)$, is also a known parameter. The only unknown quantities in Eq. (16) are C_p^* and C_{pk}^*. Substituting Eq. (16) in Eq. (15), we get the following important relation:

$$\sigma_1^2 + \sigma_2^2 + \cdots + \sigma_n^2 = \frac{T^2}{9C_p^{*2}} = \frac{d^2}{9C_{pk}^{*2}}. \tag{17}$$

The following is an important observation derived out of the Eq. (17).

Once the pair (C_p^*, C_{pk}^*) is fixed for the end-to-end lead time Y, the feasible solution set gets automatically fixed as the set of all those n-tuples $(\sigma_1, \sigma_2, \ldots, \sigma_n)$ which satisfy this equation for the chosen value of C_p^* and C_{pk}^*.

The idea behind obtaining the pair (C_p^*, C_{pk}^*) is to choose such a pair which satisfies both Constraint (12) as well as Constraint (13). By using such a pair in Eq. (17), we can get a single constraint, in terms of the decision variables, which captures both the constraints. In this way we express the constraints in terms of decision variables.

It is quite possible that multiple pairs satisfy the above requirement. In such a situation, selection of the best pair is an interesting exercise which we discuss in the next step. Moreover, there are situations when no pair satisfies the requirement. In such a situation, the VPA problem does not have any solution. Identifying such types of situations is also considered in the next section.

4.4.3. Step 3: Determining values of C_p^* and C_{pk}^*

Note that the relation (17) forces the desired (C_p^*, C_{pk}^*) pair to lie on the line $C_{pk} = \frac{d}{T} C_p$ in the C_p–C_{pk} plane. Also, it is easy to see that the Constraint (12) forces the desired pair to lie on or above the curve $C_{pm} = C_{pm}^*$ in the C_p–C_{pk} plane. Similarly, the Constraint (13) forces it to lie on or above the 6σ curve in the same plane. All these result in a feasible region in the C_p–C_{pk} plane. Depending on relative position of $C_{pm} = C_{pm}^*$ curve (call this C_{pm} curve for short) and 6σ curve (call this σ curve for short), the feasible region may take different shapes. Fig. 6 shows the geometric shapes of such a feasible region. For the purpose of analysis, we classify these geometric shapes into five different cases where we discriminate them based on the number of points at which the two curves (σ curve and C_{pm} curve) intersect each other. It is clear from Fig. 6 that the feasible region in each case is the part of the line $C_{pk} = \frac{d}{T} C_p$, denoted by OP, which intersects the shaded region. For the sake of clarity, we have shown the line OP only in Case 1. In all other cases it is understood. Each point of the feasible region satisfies both Constraints (12) and (13) and therefore can be used as a design point in Eq. (17). The concern here is which point should be selected as the design point. Before we investigate further in this direction, let us consider a few interesting facts about such a (C_p^*, C_{pk}^*) pair. In what follows is two lemmas that describes a few facts about the pair (C_p^*, C_{pk}^*).

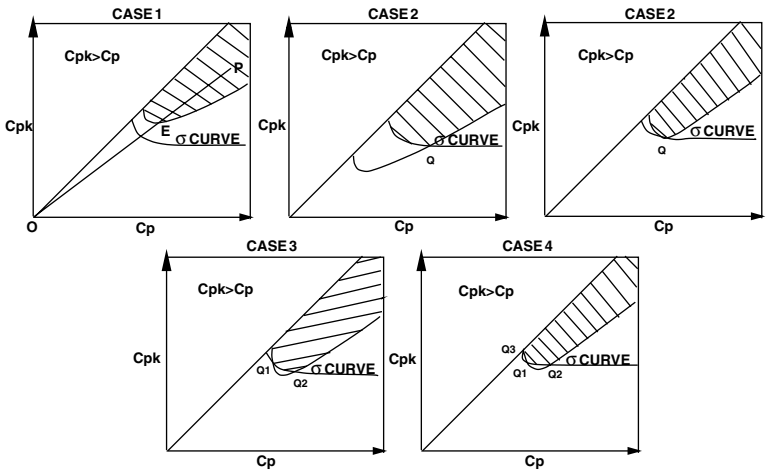

Fig. 6. Possible geometric shapes of feasible region for C_p and C_{pk} of Y.

Lemma 4.4.1. *For the given values of T and d, there is an upper bound on delivery sharpness (DS) which can be achieved for Y. This is given by*

$$\overline{C_{pm}} = \frac{T}{3(T-d)}.$$

Proof. Observe from Eq. (17) that, for given values of T and d, C_p and C_{pk} of the process Y must satisfy the following relation which is a straight line when plotted on the C_p–C_{pk} plane:

$$C_{pk} = \left(\frac{d}{T}\right) C_p. \tag{18}$$

If we take any point on this line, it represents a unique combination of C_p, C_{pk}, and C_{pm}. Hence if we choose this point as design point then the DS for Y gets fixed. Now consider the following equation for a typical C_{pm} curve on the C_p–C_{pk} plane:

$$\frac{1}{C_{pm}^2} = \frac{1}{C_p^2} + 9\left(1 - \frac{C_{pk}}{C_p}\right)^2. \tag{19}$$

It can be verified that this equation represents a hyperbola. It is quite possible that the line given by Eq. (18) becomes an asymptote of such a hyperbola. Such a hyperbola is the plot of $\overline{C_{pm}}$ because it is clear from the geometry of the figure that this line cannot intersect any other C_{pm} curve which is more than $\overline{C_{pm}}$. Hence it is not possible to achieve the C_{pm} value (or DS) higher than $\overline{C_{pm}}$ for process Y.

It is easy to show that the slope of asymptotes of $\overline{C_{pm}}$ curve is $\left(1 \pm \frac{1}{(3\overline{C_{pm}})}\right)$. Equating these to the slope of the line (18) we get

$$\overline{C_{pm}} = \frac{T}{3(T-d)}. \qquad \square$$

Lemma 4.4.2. *For fixed values of T and d,* DP *and* DS *have one-to-one correspondence with each other. Moreover,* DP *and* DS *have positive correlation.*

Proof. Earlier we stated that (C_p^*, C_{pk}^*) pair is chosen for Y in such a way that apart from satisfying both the Constraints (12) and (13), the pair must lie on the line (18). It is easy to verify that for a given point on the line (18), a unique C_{pm} curve and a unique σ curve pass through the point. This σ value and C_{pm} value are the final DP and DS respectively which are achieved for Y if this particular point is chosen as design point. Hence, it can be concluded that once a value is chosen for DP of Y, it will automatically decide the corresponding value of DS and also vice versa. To prove the other statement of the lemma, observe that as we move from point $C_p = 0$ to point $C_p = \infty$ on the line (18), the values of both C_{pm} curve and σ curve which pass through that point increase. Therefore, DP increases (or decreases) as DS increases (or decreases) for given values of T and d. $\square$

The implication of Lemma 4.4.1 is the following. If the desired C_{pm}^* is greater than $\overline{C_{pm}}$ for the given values of T and d then the problem is infeasible. In such a situation we need not proceed any further.

Lemma 4.4.2 also has a key implication on the problem of fixing the values of C_p^* and C_{pk}^* for Y. According to Lemma 4.4.2, DP and DS of Y get fixed immediately as soon as a feasible point from the line, given by Eq. (18), is chosen as design point. It is easy to see that each point on the C_p–C_{pk} plane is unique in its own because it has a unique combination of DP and DS. Therefore, it is quite possible that the point which we have chosen results in either higher DP or higher DS than required for the end-to-end delivery process. Thus it is not always true that the DP and DS for Y obtained from the design will exactly be the same as specified in the Constraints (12) and (13).

In view of the above findings, the problem of fixing the values of C_p^* and C_{pk}^* can be addressed in following way. First step is to test the feasibility of the problem by means of Lemma 4.4.1. If the problem turns out to be feasible then each point in the feasible region is eligible to be selected as a design point. However, depending upon the point which is chosen as design point, the final cost $\mathscr{K}^*$ (which we get out of solving the optimization problem) may vary. At this point, we cannot say which feasible point will result in minimum cost. Hence, the problem is handled in an indirect manner. The proposed scheme is like this. First solve the optimization problem without any constraint and get the optimal variance σ^g for Y. It will result in global minimum cost. Now use this variance σ^g to get C_p^g and C_{pk}^g for Y which result in minimum cost. If the point (C_p^g, C_{pk}^g) falls in the feasible region then this point is used as a design point (C_p^*, C_{pk}^*), otherwise the point E where the line OP enters into the shaded region is taken as the desired (C_p^*, C_{pk}^*) pair. The reason behind choosing the point E as design point is following. The values DP and DS which result from point E are minimum possible values satisfying both the Constraints (12) and (13). If we choose any other feasible point then even though the resulting DP and DS for Y will satisfy the Constraints (12) and (13), yet their values will be a bit high and this will lead to higher cost. The point E can be determined by solving the corresponding equation of the plots. A detailed algorithm for determining the point E is discussed in [39]. In this way we obtain the constraints in terms of decision variables.

4.4.4. Step 4: Solving the optimization problem

The optimization problem which we presented in Step 1 can now be written as:

Minimize

$$\mathscr{K} = \sum_{i=1}^{n} \mathscr{K}_i = \sum_{i=1}^{n} (A_{i0} + A_{i1}\sigma_i + A_{i2}\sigma_i^2) \tag{20}$$

subject to

$$\sum_{i=1}^{n} \sigma_i^2 = \frac{T^2}{9C_p^{*2}} = \frac{d^2}{9C_{pk}^{*2}}, \tag{21}$$

$$\sigma_i > 0 \quad \forall i. \tag{22}$$

This is a nonlinear optimization problem with an equality constraint. Therefore, we can use the Lagrange multiplier method to compute the stationary points. After getting the stationary points, we can apply the sufficiency condition to determine the points of minima. Here we just present the formulation for finding the stationary points and leave the remaining part to be covered in a detailed example.

Method of Lagrange multipliers applied to the VPA problem.

1. *Lagrange function*

The Lagrange function $L(\sigma_1, \sigma_2, \ldots, \sigma_n, \lambda)$ is given by:

$$L(\sigma_1, \sigma_2, \ldots, \sigma_n, \lambda) = \mathscr{K} + \lambda\left(\sum_{i=1}^{n} \sigma_i^2 - \sigma^{*2}\right), \quad \text{where } \sigma^{*2} = \frac{T^2}{9C_p^{*2}} = \frac{d^2}{9C_{pk}^{*2}}. \tag{23}$$

2. *Necessary condition for stationary points*

If the point $\mathscr{P}^* = (\sigma_1^*, \sigma_2^*, \ldots, \sigma_n^*, \lambda^*)$ happens to be the optimal point, then this point must satisfy the following necessary conditions:

$$\left.\frac{\partial L}{\partial \sigma_1}\right|_{\mathscr{P}^*} = \left.\frac{\partial L}{\partial \sigma_2}\right|_{\mathscr{P}^*} = \cdots = \left.\frac{\partial L}{\partial \sigma_n}\right|_{\mathscr{P}^*} = \left.\frac{\partial L}{\partial \lambda}\right|_{\mathscr{P}^*} = 0.$$

Substituting the value of Lagrange function L from Eq. (23), we get the following necessary conditions:

$$\left.\frac{\partial L}{\partial \sigma_1}\right|_{\mathscr{P}^*} = A_{11} + 2A_{12}\sigma_1^* + 2\lambda^*\sigma_1^* = 0,$$
$$\left.\frac{\partial L}{\partial \sigma_2}\right|_{\mathscr{P}^*} = A_{21} + 2A_{22}\sigma_2^* + 2\lambda^*\sigma_2^* = 0,$$
$$\cdots$$
$$\left.\frac{\partial L}{\partial \sigma_n}\right|_{\mathscr{P}^*} = A_{n1} + 2A_{n2}\sigma_n^* + 2\lambda^*\sigma_n^* = 0,$$
$$\left.\frac{\partial L}{\lambda}\right|_{\mathscr{P}^*} = \sum_{i=1}^{n}(\sigma_i^{*2}) - \sigma^{*2} = 0.$$

Solving the above system of equations will give us all the stationary points. After discarding those stationary points that do not satisfy Constraint (22), we will be left with the stationary points that satisfy both the constraints. It is required now to apply the sufficiency condition in order to find out the points of minima. We omit this for the sake of brevity.

4.4.5. Step 5: Solving the specific decision problem

We present the discussion for the case of the optimal partner selection problem. Without loss of generality, let there be n stages in the supply chain and at each individual stage, let there be several alternatives available (for example, if it is a logistics stage, we have several logistics providers). Each of these alternatives has a standard deviation associated with the promised lead time. Now the problem is to choose one service provider for each stage out of the given alternatives. It is not difficult to see that each combination of the partners will result in a unique DP, DS, and delivery cost for the end-to-end delivery process. We wish to choose the combination which meets the given standards for DP and DS in a minimum possible cost. Thus the problem is highly combinatorial in nature. The brute force technique of finding the solution involves computing the DP, DS, and end-to-end delivery cost for each combination and then picking up the optimal one. Note that the complexity of such a technique would be of the order of $O(N_1 \times N_2 \times \cdots \times N_n \times \xi)$, where N_i; $i = 1, 2, \ldots, n$, is the number of alternative service providers for the stage i and ξ is the complexity involved in computing the DP, DS, and end-to-end delivery cost for one combination of the partners which can shown to be the constant. The exponential complexity of the exhaustive enumeration technique motivates us to seek a better algorithm. Here, we present an efficient algorithm whose complexity turns out to be $O(2^n\xi + \psi)$, where ψ is the computation time involved in solving one instance of the VPA problem. ψ can be easily shown to be linear in terms of number of stages. The idea behind this algorithm is following.

First execute Steps 1–4 on the underlying VPA problem and get $\sigma_1^*, \sigma_2^*, \ldots, \sigma_n^*$. Now consider stage i. Out of all the available service providers for stage i, choose two service providers L_i and R_i such that the variance of processing time when the work is done by these two are σ_i^l, and σ_i^r respectively and these values are immediately below and immediately above σ_i^*. Note that it may happen that σ_i^* is less than all candidate standard deviations, in which case, we choose the one that is closest to σ_i^* (this will reduce the computational complexity even further).

After fixing two alternatives L_i and R_i for each stage i, we will be left with 2^n ways in which we can choose a mix of alternatives for the supply chain. For each possible mix, we can compute the total cost $\mathscr{K}$ by using relation (11). The σ for Y can be computed by using relation (15) where σ_i will now be replaced by either σ_i^l or σ_i^r depending upon which service provider we have chosen. This σ can be used to compute C_p and C_{pk} for end-to-end lead time Y through the relation (16). After computing these values it is easy to decide the optimal combinations of service providers all along the supply chain. Note that the worst case

number of combinations that need to be considered is 2^n, independent of the number of alternatives available at each individual stage.

Two remarks regarding the above algorithm are in order.

- Note that $O(2^n\xi + \psi)$ is a significant improvement over $O(N_1 \times N_2 \times \cdots \times N_n \times \xi)$. It ensures that the complexity of the partner selection algorithm is dependent only on the number of stages and is independent of the number of service providers at each stage of the supply chain.
- It is easy to see that in the event a supply chain partner drops out or a new supply chain partner enters the fray, the proposed algorithm handles the changes with ease. In the case of a new entry, all that we have to do is just compute DP, DS, and delivery cost for all the $2^{(n-1)}$ new combinations of the supply chain partners that will emerge due to this new entry. Now check if any one of this new mix does better than the current optimal mix. Similarly, if any partner drops out then we need to do nothing if the leaving partner is not a member of the current set of optimal partners. Otherwise, we can just delete all those combinations which include this member as one of the partners and determine the optimal one out of the remaining 2^n–$2^{(n-1)}$ combinations.
- If in one stage, say stage i, the only available alternatives are far from the ideal solution σ_i^* it may be beneficial to compensate by choosing an alternative in another stage j far from σ_j^* (i.e. not necessarily the alternatives closest to the local optimum) in order to satisfy the required condition in the cheapest way. In such cases, the method does not provide an optimal solution. Thus the method is to be viewed as a heuristic that provides optimal values in most situations.

5. A plastic industry case study

We now consider a supply chain for a plastics industry (a certain anonymous firm in the western state of Maharashtra, India) and apply the 5-Step approach for formulating and solving the partner selection problem. Fig. 7 depicts the supply chain at an aggregate level. The supply chain has six business processes namely (1) procurement, (2) sheet fabrication, (3) inbound logistics, (4) manufacturing, (5) assembly, and (6) outbound logistics. Let all the business processes in the supply chain satisfy the assumptions mentioned in Section 4.2. Assume for the sake of convenience that there are three alternatives (call them service providers) at each of the six stages.

The problem here is to determine the optimal mix of service providers for each stage such that the end to end delivery probability is at least at 6σ level and delivery sharpness is at least, say, 1.4. Suppose for each stage, the mean lead time for all the three alternative service providers is same. Let the mean μ_i for $i = 1, 2, 3, 4, 5$ and 6 be 7 days, 30 days, 3 days, 30 days, 10 days, and 3 days respectively. Let the target value of supply chain lead time Y be 82 days and tolerance be 6.5 days. This implies:

$$\tau = 82\,\text{days},$$

$$T = 6.5\,\text{days}.$$

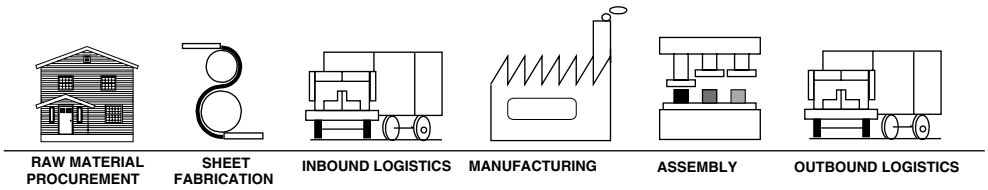

Fig. 7. An example of a linear supply chain: A typical plastic industry supply chain.

Table 1
Standard deviation of lead times and cost for each service provider

Stage	Service providers					
i	A		B		C	
	σ_{Ai} (days)	C_{Ai} (\$/item)	σ_{Bi} (days)	C_{Bi} (\$/item)	σ_{Ci} (days)	C_{Ci} (\$/item)
1	0.50	256.18	0.75	161.44	1.00	125.69
2	0.40	436.40	0.50	413.03	0.60	390.37
3	0.10	077.79	0.20	049.33	0.30	034.81
4	0.40	436.40	0.50	413.03	0.60	390.37
5	0.50	185.79	1.00	080.34	1.50	065.27
6	0.10	077.79	0.20	049.33	0.30	034.81

The processing cost of one unit of product at each one of the six business processes, varies over the service providers as a function of variance of lead times. Table 1 gives the values of per unit processing cost and processing time variance for each service provider.

5.1. Step 1

Some of the parameters for VPA problem are provided explicitly in the given problem. The parameters which will be needed in further calculations and are implicit to the problem are μ, d, θ, C^*_{pm}, and A_{ij}. If we denote the lead time distributions of procurement, sheet fabrication, inbound logistics, manufacturing, assembly, and outbound logistics by X_1, X_2, X_3, X_4, X_5, and X_6 respectively, then it is easy to see that

$$\mu = \sum_{i=1}^{6} \mu_i = 83\,\text{days},$$
$$d = \min(\tau + T - \mu, \mu - \tau + T) = 5.5\,\text{days},$$
$$\theta = 6,$$
$$C^*_{pm} = 1.4.$$

The coefficients A_{ij} can be obtained by fitting a second order polynomial curve for the given three pairs (σ_{Ai}, C_{Ai}), (σ_{Bi}, C_{Bi}), and (σ_{Ci}, C_{Ci}) for each stage i. The coefficients A_{ij} obtained by such an approximation are tabulated in Table 2.

Now the optimization problem can be formulated as follows:

Minimize

$$\mathscr{K} = \sum_{i=1}^{6} \mathscr{K}_i = \sum_{i=1}^{6} (A_{i0} + A_{i1}\sigma_i + A_{i2}\sigma_i^2) \tag{24}$$

subject to

$$\text{DS for end-to-end lead time} \geqslant 1.4,$$
$$\text{DP for end-to-end lead time} \geqslant 6\sigma,$$
$$\sigma_i > 0, \quad i = 1, 2, \ldots, 6.$$

5.2. Step 2

The constraints of the optimization problem presented in Step 1 can be expressed in terms of decision variables by invoking relation (17). This leads to

Table 2
Cost coefficients for plastic industry supply chain problem

Stage	$A_{i_0}(\frac{\$}{\text{item}})$	$A_{i_1}(\frac{\$}{\text{item-day}})$	$A_{i_2}(\frac{\$}{\text{item-day}^2})$
Procurement	622.634	−968.872	471.928
Sheet fabrication	537.011	−265.752	035.604
Inbound logistics	120.186	−493.651	696.919
Manufacturing	537.011	−265.752	035.604
Assembly	381.625	−482.053	180.770
Outbound logistics	120.186	−493.651	696.919

$$\sum_{i=1}^{6} \sigma_i^2 = \frac{42.25}{9C_p^{*2}} = \frac{30.25}{9C_{pk}^{*2}}, \tag{25}$$

$$\sigma_i > 0 \quad \forall i = 1, 2, \ldots, 6. \tag{26}$$

5.3. Step 3

As discussed earlier, the first step towards fixing the values of C_p^* and C_{pk}^* is to test the feasibility of the problem. It is easy to see $\overline{C_{pm}}$ for this problem is 2.1667 which is greater than 1.4. Therefore, the problem is feasible. As a next step, we solve the corresponding unconstrained optimization problem and get the point (C_p^g, C_{pk}^g) which leads to global minimum cost and then test whether this point falls into feasible region or not.

For this, let $S = \{(\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6) : \sigma_i \in \Re^+ \quad \forall i = 1, 2, 3, 4, 5, 6\}$. It immediately follows from this definition of S that $\mathscr{K} : S \mapsto \Re$ where S is a nonempty open convex set. To test the convexity of function $\mathscr{K}$, we compute the gradient vector and Hessian matrix of the function. Note that the gradient vector $\nabla\mathscr{K}(\overline{\mathbf{X}})$ for function $\mathscr{K}$ at point $\overline{\mathbf{X}} = (\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6)^{\mathrm{T}}$ can be given by

$$\nabla\mathscr{K}(\overline{\mathbf{X}}) = \begin{bmatrix} \frac{\partial\mathscr{K}(\overline{\mathbf{X}})}{\partial\sigma_1} \\ \frac{\partial\mathscr{K}(\overline{\mathbf{X}})}{\partial\sigma_2} \\ \frac{\partial\mathscr{K}(\overline{\mathbf{X}})}{\partial\sigma_3} \\ \frac{\partial\mathscr{K}(\overline{\mathbf{X}})}{\partial\sigma_4} \\ \frac{\partial\mathscr{K}(\overline{\mathbf{X}})}{\partial\sigma_5} \\ \frac{\partial\mathscr{K}(\overline{\mathbf{X}})}{\partial\sigma_6} \end{bmatrix} = \begin{bmatrix} A_{11} + 2A_{12}\sigma_1 \\ A_{21} + 2A_{22}\sigma_1 \\ A_{31} + 2A_{32}\sigma_1 \\ A_{41} + 2A_{42}\sigma_1 \\ A_{51} + 2A_{52}\sigma_1 \\ A_{61} + 2A_{62}\sigma_1 \end{bmatrix}.$$

Also, the Hessian $H(\overline{\mathbf{X}})$ for function $\mathscr{K}$ at point $\overline{\mathbf{X}} = (\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6)^{\mathrm{T}}$ turns out be

$$H(\overline{\mathbf{X}}) = \begin{bmatrix} 2A_{12} & 0 & 0 & 0 & 0 & 0 \\ 0 & 2A_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & 2A_{32} & 0 & 0 & 0 \\ 0 & 0 & 0 & 2A_{42} & 0 & 0 \\ 0 & 0 & 0 & 0 & 2A_{42} & 0 \\ 0 & 0 & 0 & 0 & 0 & 2A_{42} \end{bmatrix}.$$

Observe that the gradient vector and Hessian exist for each $\overline{\mathbf{X}} \in S$. Hence function $\mathscr{K}$ is twice differentiable over open convex set S. Moreover, the Hessian is independent of $\overline{\mathbf{X}}$. Therefore, it is enough that we test the positive definiteness (PD) or positive semi definiteness (PSD) of Hessian at any point of S instead of testing it over whole S.

It is easy to see that all the diagonal elements of Hessian are positive real numbers because A_{i2} are positive real numbers. Therefore, the Hessian is PD and the function $\mathscr{K}$ is strictly convex. It implies that a local minimum of function $\mathscr{K}$ is the unique global minimum. This can be obtained by equating $\nabla\mathscr{K}(\overline{\mathbf{X}})$ to 0. The optimal values of variance for the stages come out be $\sigma_1^g = 1.0265$ days, $\sigma_2^g = 3.732$ days, $\sigma_3^g = 0.3541$ days, $\sigma_4^g = 3.732$ days, $\sigma_5^g = 1.3333$ days and $\sigma_6^g = 0.3541$ days. The corresponding variance of end-to-end lead time Y comes out to be $\sigma^g = 5.5622$ days. Also $C_p^g = 0.3895$ and $C_{pk}^g = 0.3296$. In order to test the feasibility of this point (C_p^g, C_{pk}^g) first we compute the DP and DS values which will be obtained if this point is chosen as design point. According to Lemma 4.4.2, the C_{pm} curve and σ curve which pass through it are those desired DP and DS. These values comes out to be $\mathrm{DP} = 2.17393\sigma$ and $\mathrm{DS} = 0.383359$.

Because these values are less than what are desired i.e. $\mathrm{DP} = 6\sigma$ and $\mathrm{DS} = 1.4$, the point (C_p^g, C_{pk}^g) cannot be taken as design point and we will have to use point E (the point of intersection of the line OP and feasible region) as design point. The current problem falls in Case 2 (Subcase A) of Fig. 6. Therefore point $E(C_p^*, C_{pk}^*)$ can be computed by solving the Eqs. (19) and (18). This point comes out to be:

$$C_p^* = 1.834364282,$$

$$C_{pk}^* = 1.552154393.$$

The DP and DS which are obtained for Y by using this point as design point are 6.15645σ and 1.4 respectively.

5.4. Step 4

Substituting the values of C_p^*, C_{pk}^* in Eq. (25), we obtain the following constraint to work with while solving the optimization problem:

$$\sum_{i=1}^{6} \sigma_i^2 = 1.389060165.$$

Now we will apply the Lagrange multiplier method to solve this optimization problem.

1. *Lagrange function*

Lagrange function $L(\sigma_1, \sigma_2, \ldots, \sigma_6, \lambda)$ is given as:

$$L(\sigma_1, \sigma_2, \ldots, \sigma_6, \lambda) = \mathscr{K} + \lambda\left(\sum_{i=1}^{6} \sigma_i^2 - 1.389060165\right).$$

2. *Necessary condition for stationary points*

Let point $\mathscr{P}^* = (\sigma_1^*, \sigma_2^*, \ldots, \sigma_6^*, \lambda^*)$ correspond to the optimal point, then this point must satisfy the following necessary conditions:

$$
\begin{aligned}
&-968.872 + 2(471.928)\sigma_1^* + 2\lambda^*\sigma_1^* = 0,\\
&-265.752 + 2(035.604)\sigma_2^* + 2\lambda^*\sigma_2^* = 0,\\
&-493.651 + 2(696.919)\sigma_3^* + 2\lambda^*\sigma_3^* = 0,\\
&-265.752 + 2(035.604)\sigma_4^* + 2\lambda^*\sigma_4^* = 0,\\
&-482.053 + 2(180.770)\sigma_5^* + 2\lambda^*\sigma_5^* = 0,\\
&-493.651 + 2(696.919)\sigma_6^* + 2\lambda^*\sigma_6^* = 0,\\
&\sigma_1^{*2} + \sigma_2^{*2} + \sigma_3^{*2} + \sigma_4^{*2} + \sigma_5^{*2} + \sigma_6^{*2} = 1.389060165.
\end{aligned}
$$

Solving this system of equations by standard numerical methods we get the following solutions:

$$
\begin{aligned}
&\sigma_1^* = 0.680498\,\text{days},\\
&\sigma_2^* = \sigma_4^* = 0.482201\,\text{days},\\
&\sigma_3^* = \sigma_6^* = 0.263456\,\text{days},\\
&\sigma_5^* = 0.572881\,\text{days}.
\end{aligned}
$$

Under this operating condition, the cost of delivery is:

$$\mathscr{K}^* = 802.299\,\frac{\$}{\text{item}}.$$

It can be verified easily by the sufficiency condition that this point indeed corresponds to the point of minima.

5.5. Step 5

By comparing the optimal standard deviations σ_i^* obtained in Step 4 with the given data in Table 1 we can compute, for each stage, the service providers whose variance is closest to the optimal. These are listed in Table 3. It is easy to see that we can construct 64 combinations out of these 12 service providers listed in Table 3, where each combination representing a particular mix of service providers. We have computed the end-to-end supply chain cost $\mathscr{K}$, process capability indices C_p and C_{pk}, DP, and DS for each of these 64 combination and the results are tabulated in Table 4. In this table, for each combination, rather than computing the exact sigma level for DP we have only specified whether or not DP is greater than 6σ level. If greater, we have indicated in the corresponding by 'Y', otherwise it is indicated by 'N' (for "no").

From Table 4, it can be easily seen that the combination which ensures the desired DP and DS level in minimum possible cost is the combination number 53 i.e. *BBBBAB*. Thus the optimal mix of service providers for stages 1, 2, 3, 4, 5, and 6 of the given problem are *B*, *B*, *B*, *B*, *A*, and *B* respectively.

Table 3
Pairs of almost optimal service providers for each stage

Stage (i)	L_i	R_i
1	*A*	*B*
2	*A*	*B*
3	*B*	*C*
4	*A*	*B*
5	*A*	*B*
6	*B*	*C*

Table 4
DP, DS, and delivery cost for combinations of near optimal service providers for each stage of plastic industry supply chain

Combination		C_p	C_{pk}	DP (6σ level)	DS	$\mathscr{K}$ (\$/unit)
1	*AABAAB*	2.283867	1.932503	N	1.571866	1413.429932
2	*AABAAC*	2.222953	1.880960	Y	1.551582	1398.910034
3	*AABABB*	1.686748	1.427248	Y	1.330972	1307.979980
4	*AABABC*	1.661757	1.406102	N	1.318591	1293.459961
5	*AABBAB*	2.177582	1.842569	N	1.535908	1390.059937
6	*AABBAC*	2.124591	1.797731	Y	1.516970	1375.540039
7	*AABBBB*	1.642546	1.389846	N	1.308930	1284.609985
8	*AABBBC*	1.619443	1.370298	Y	1.297150	1270.089966
9	*AACAAB*	2.222953	1.880960	Y	1.551582	1398.910034
10	*AACAAC*	2.166667	1.833333	N	1.532063	1384.390015
11	*AACABB*	1.661757	1.406102	N	1.318591	1293.459961
12	*AACABC*	1.637846	1.385870	N	1.306550	1278.939941
13	*AACBAB*	2.124591	1.797731	Y	1.516970	1375.540039
14	*AACBAC*	2.075290	1.756015	N	1.498715	1361.020020
15	*AACBBB*	1.619443	1.370298	Y	1.297150	1270.089966
16	*AACBBC*	1.597288	1.351551	Y	1.285680	1255.569946
17	*ABBAAB*	2.177582	1.842569	N	1.535908	1390.059937
18	*ABBAAC*	2.124591	1.797731	Y	1.516970	1375.540039
19	*ABBABB*	1.642546	1.389846	N	1.308930	1284.609985
20	*ABBABC*	1.619443	1.370298	Y	1.297150	1270.089966
21	*ABBBAB*	2.084876	1.764126	Y	1.502313	1366.689941
22	*ABBBAC*	2.038229	1.724655	Y	1.484575	1352.170044
23	*ABBBBB*	1.601646	1.355239	N	1.287950	1261.239990
24	*ABBBBC*	1.580204	1.337096	Y	1.276721	1246.719971
25	*ABCAAB*	2.124591	1.797731	Y	1.516970	1375.540039
26	*ABCAAC*	2.075290	1.756015	N	1.498715	1361.020020
27	*ABCABB*	1.619443	1.370298	Y	1.297150	1270.089966
28	*ABCABC*	1.597288	1.351551	Y	1.285680	1255.569946
29	*ABCBAB*	2.038229	1.724655	Y	1.484575	1352.170044
30	*ABCBAC*	1.994578	1.687720	Y	1.467452	1337.650024
31	*ABCBBB*	1.580204	1.337096	Y	1.276721	1246.719971
32	*ABCBBC*	1.559601	1.319662	N	1.265780	1232.199951
33	*BABAAB*	1.967665	1.664948	Y	1.456635	1318.689941
34	*BABAAC*	1.928308	1.631645	Y	1.440448	1304.170044
35	*BABABB*	1.546633	1.308689	N	1.258817	1213.239990
36	*BABABC*	1.527299	1.292330	N	1.248327	1198.719971
37	*BABBAB*	1.898468	1.606396	N	1.427882	1295.319946
38	*BABBAC*	1.863046	1.576423	N	1.412625	1280.800049
39	*BABBBB*	1.512344	1.279676	N	1.240122	1189.869995
40	*BABBBC*	1.494253	1.264368	Y	1.230088	1175.349976
41	*BACAAB*	1.928308	1.631645	Y	1.440448	1304.170044
42	*BACAAC*	1.891222	1.600264	Y	1.424790	1289.650024
43	*BACABB*	1.527299	1.292330	N	1.248327	1198.719971
44	*BACABC*	1.508673	1.276569	N	1.238094	1184.199951
45	*BACBAB*	1.863046	1.576423	N	1.412625	1280.800049
46	*BACBAC*	1.829535	1.548068	Y	1.397849	1266.280029
47	*BACBBB*	1.494253	1.264368	Y	1.230088	1175.349976
48	*BACBBC*	1.476796	1.249597	N	1.220295	1160.829956
49	*BBBAAB*	1.898468	1.606396	N	1.427882	1295.319946
50	*BBBAAC*	1.863046	1.576423	N	1.412625	1280.800049
51	*BBBABB*	1.512344	1.279676	N	1.240122	1189.869995
52	*BBBABC*	1.494253	1.264368	Y	1.230088	1175.349976
53	*BBBBAB*	1.836092	1.553616	Y	1.400767	1271.949951
54	*BBBBAC*	1.803990	1.526453	N	1.386356	1257.430054

Table 4 (*continued*)

Combination		C_p	C_{pk}	DP (6σ level)	DS	$\mathscr{K}$ (\$/unit)
55	*BBBBBB*	1.480238	1.252509	N	1.222234	1166.500000
56	*BBBBBC*	1.463263	1.238145	Y	1.212624	1151.979980
57	*BBCAAB*	1.863046	1.576423	N	1.412625	1280.800049
58	*BBCAAC*	1.829535	1.548068	Y	1.397849	1266.280029
59	*BBCABB*	1.494253	1.264368	Y	1.230088	1175.349976
60	*BBCABC*	1.476796	1.249597	N	1.220295	1160.829956
61	*BBCBAB*	1.803990	1.526453	N	1.386356	1257.430054
62	*BBCBAC*	1.773515	1.500667	Y	1.372381	1242.910034
63	*BBCBBB*	1.463263	1.238145	Y	1.212624	1151.979980
64	*BBCBBC*	1.446858	1.224264	Y	1.203239	1137.459961

Note that Table 4 can only have at most 64 entries. Often, if a service provider is already fixed for a particular stage, this number will be much less than 64. Also, note that whatever the number of candidate service providers at each stage, we will have to look into at most 64 combinations in this case because of the variance pool allocation that we have already done.

5.6. Some more insights

A sensitivity analysis of the VPA problem is possible if we take different points on the line (18), which is $C_{pk} = \frac{6.5}{5.5} C_p$ in this case, and solve the optimization problem. We have solved this problem for a couple of points and tried to investigate the dependencies of optimal cost on DS. The plot is shown in Fig. 8. Several inferences can be deduced from this plots.

- A given DP results in a unique DS and also vice versa. Therefore, the DP and DS which are given as constraints in the problem may not be a valid pair and cannot be achieved as it is. However, VPA tries to choose a pair of DP and DS which suit the requirement in the best possible manner. For example, from the above plot it is clear that DS corresponding to DP $= 6\sigma$ is less than 1.4, therefore the point for which DS $= 1.4$ is chosen as design point, even though the corresponding DP $= 6.15645\sigma$ is a little higher than 6σ. The reason is that this point suits the design requirements in the best possible manner.

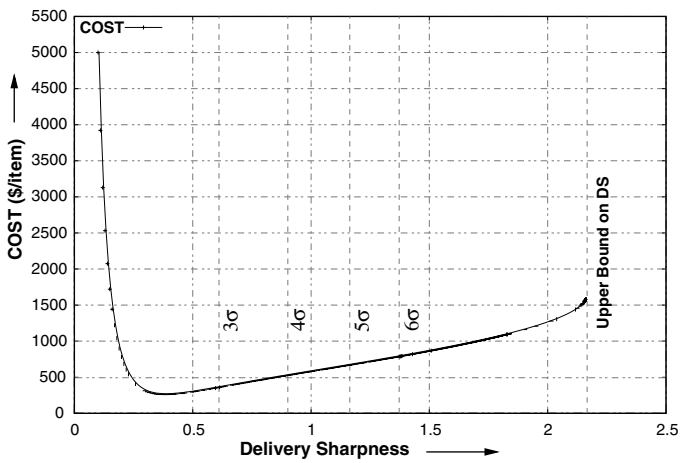

Fig. 8. Effect of delivery quality on delivery cost.

Table 5
Sample values for decision variables at some representative DP and DS pairs

DP	DS	σ_1 (days)	$\sigma_2 = \sigma_4$ (days)	$\sigma_3 = \sigma_6$ (days)	σ_5 (days)	Cost (\$/item)
6σ	1.37214	0.694302	0.508311	0.267497	0.592825	785.952
5σ	1.16366	0.788656	0.746783	0.294104	0.745988	668.365
4σ	0.90195	0.885917	1.202550	0.319801	0.942760	525.029
3σ	0.61113	0.971198	2.126740	0.341016	1.160770	358.195

- The curve can be divided into two parts
  - From $C_{pm} = 0$ to the $C^g_{pm} = 0.383359$ (point of global minima). In this part delivery cost decreases as the quality of delivery increases.
  - From $C^g_{pm} = 0.383359$ to $\overline{C_{pm}} = 2.1667$. In this part delivery cost increases as the quality of delivery increases.

  The behavior of the second part of the curve is consistent with our intuition but the first part is counter-intuitive. A justification behind this is as follows. As described earlier, for a given actual yield α, there exist upper bounds and lower bounds for C_p and C_{pk}. In a similar way it is possible to get the lower bound for C_{pm} value also. It implies that in order to achieve a specified level of precision, a minimum level of accuracy is a must. If accuracy of the process is lower than that minimum level, then no matter how much effort one puts in, the precision can never reach the specified level. In one sentence it can be summarized as "*For being precise*, *one should be accurate also*". This is the fundamental cause behind the behavior of the curve in its first part. In the first part of the curve, accuracy is so low that even achieving a relatively low precision itself is very costly but in the second part of the curve accuracy is so high that achieving such high accuracy itself is very expensive.
- Table 5 lists some sample values of decision variables σ_i and optimal delivery costs at some representative DP values along with corresponding DS values.

The following is an interesting observation made through these sample values. As the quality level increases, variance of end-to-end lead time Y (i.e. σ) decreases. In order to accommodate such reduction in σ, variance σ_i of individual process(es) reduces. Observe that the processes which are expensive, for example procurement, undergo a very little change in variance. However, the cheaper processes, such as sheet fabrication and manufacturing, are used as a vehicle to reduce the variance. The reason behind it is reducing the variance of cheaper processes is more cost effective than reducing the variance of expensive processes for achieving the same quality level of end-to-end lead time.

6. Summary and future work

In this paper, we have presented an approach to achieve variability reduction, synchronization, and therefore delivery performance improvement in supply chain networks. Our approach exploits connections between design tolerancing in mechanical assemblies and lead time compression in supply chain networks. The specific problem we solved here is the variance pool allocation problem. The VPA problem distributes a pool of variance across individual stages of a supply chain in a cost effective way, so as to achieve desired levels of delivery performance.

The contributions of this paper can be summarized as follows.

- Explaining the relevance of process capability indices C_p, C_{pk}, and C_{pm} in describing variability effects in the end-to-end supply chain delivery process.

- Introducing two performance metrics, delivery probability and delivery sharpness, to describe the precision and accuracy aspects of supply chain deliveries.
- Formulating the variance pool allocation ion problem, an important design optimization problem in supply chains.
- Proposing a five step approach to solve the VPA problem for linear supply chains.
- Illustrating the efficacy of the approach through a six stage plastics industry case study, by solving a specific problem, namely choosing an optimal mix of service providers in the supply chain stages.

At this stage, we would like to make an important observation about the proposed approach. The models of the type discussed here are best suited for first level decision making which throws up several candidate solutions and rejects unsatisfactory candidates. For example, we can choose to reject all candidates who fail to achieve, say, a three sigma level of delivery performance. Subsequently, at the second level of decision making, the candidate solutions shortlisted at the first level are shrunk based on strategic concerns, relationships, and the like. It is important to have a shortlisting mechanism based on sound models and considerations and the proposed work is positioned to be used here. It would not be prudent to use this model alone to take important strategic decisions such as supplier selection.

6.1. Future work

The paper leaves plenty of room for further work in several directions. The VPA problem has been investigated only for linear supply chains. Apart from computational reasons, there is no major difficulty in solving the VPA problem for supply chains with non-linear flows. VPA is only one of a rich variety of design optimization problems that one can address in the framework developed in this paper. An immediate problem that would strike one here is allocation of a pool of nominals among individual business processes. This has implication for choosing resources such as logistics and suppliers in an optimal way. Choice of an optimal mix of customer orders is another problem that could be attempted in this framework. In a companion article [37,41], we have looked at an inventory allocation problem in a multi-echelon supply chain, where we use the framework (variance pool allocation) developed in this paper to determine optimal inventory levels in different supply chain stages.

In case the various supply chain partners have different expected lead times, which is likely in practice, the proposed solution procedure cannot be used as is. We can certainly formulate the problem so as to relax this assumption. The resulting problem will be two dimensional (that is, involving allocation of both means and variabilities) and under quadratic approximation, should pose no difficulties in solving. This is an interesting extension to the problem.

The supply chain example that we have looked at belongs to the MTO type. Here again, there is no reason why our approach cannot be applied for coordination types other than MTO, such as MTS and BTO (Build to Order). In fact, in the framework that we have developed in this paper, one can address almost any type of design optimization problem with variability reduction as the basic strategy. In a related paper [37] and a Master's thesis [39], we have defined the general notion of a *six sigma supply chain* and presented a general mathematical programming problem for supply chain design optimization.

Finally, variability is certainly not lead time alone. Variation is fundamental to any metric or process. In this paper, we have emphasized lead time (motivated by the importance of time based competition). End-to-end lead time is an encompassing metric that takes into account most aspects of system dynamics (for example, resource contention, queuing, inventories, etc.). The framework discussed in the paper will apply equally well to any metric other than lead time too. In general, let $X_1, \ldots, X_n$ represent n random variables that describe n phenomena in any system and let Y be a performance metric of interest, which is given as, $Y = f(X_1, \ldots, X_n)$, where f is a deterministic function (analytic or computational). The framework developed here will apply to any Y, as long as f is known deterministically.

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Appendix A. Proof of the results in (5)–(8):

Proof for (5): We start with the equation $T = b + d$ and it is easy to see that

$$\begin{aligned} T &= b + d \\ &\Rightarrow 1 = \frac{b}{T} + \frac{d}{T} \\ &\Rightarrow 1 = \frac{b}{T} + \left(\frac{d}{3\sigma}\right)\left(\frac{3\sigma}{T}\right) \\ &\Rightarrow 1 = \frac{b}{T} + \frac{C_{pk}}{C_p} \\ &\Rightarrow C_{pk} = C_p\left(1 - \frac{b}{T}\right). \end{aligned}$$

Proof for (6): Rearrangement of expression for C_{pm} results in

$$\begin{aligned} &\frac{1}{9C_{pm}^2} = \frac{\sigma^2}{T^2} + \frac{b^2}{T^2} \\ &\Rightarrow \frac{1}{9C_{pm}^2} = \frac{1}{9}\left(\frac{3\sigma}{T}\right)^2 + \left(\frac{b}{T}\right)^2. \end{aligned}$$

Substituting the values from Eqs. (1) and (5), we get the desired identity relation:

$$\frac{1}{9C_{pm}^2} = \frac{1}{9C_p^2} + \left(1 - \frac{C_{pk}}{C_p}\right)^2.$$

Proof for (7): By the definition of *potential*, it is easy to see that

$$\begin{aligned} \text{Potential} &= \Phi\left(\frac{U-\mu}{\sigma}\right) - \Phi\left(\frac{L-\mu}{\sigma}\right), \\ &= 2\left(\Phi\left(\frac{U-\mu}{\sigma}\right) - 0.5\right), \\ &= 2\left(\Phi\left(\frac{T}{\sigma}\right) - 0.5\right), \\ &= 2\Phi\left(3\left(\frac{T}{3\sigma}\right)\right) - 1, \\ &= 2\Phi(3C_p) - 1. \end{aligned}$$

Proof for (8): We will prove the result for the case $\tau > \mu$. The arguments will be symmetric in the case $\tau \leqslant \mu$. It is easy to see that

$$\begin{aligned}
\text{Actual yield} &= \Phi\left(\frac{U-\mu}{\sigma}\right) - \Phi\left(\frac{L-\mu}{\sigma}\right), \\
&= \Phi\left(\frac{d}{\sigma}\right) - \left(1 - \Phi\left(\frac{\mu - L}{\sigma}\right)\right), \\
&= \Phi\left(\frac{d}{\sigma}\right) - 1 + \Phi\left(\frac{b+T}{\sigma}\right), \\
&= \Phi\left(\frac{3d}{3\sigma}\right) + \Phi\left(\frac{2T-d}{\sigma}\right) - 1, \\
&= \Phi\left(3C_{pk}\right) + \Phi\left(6C_p - 3C_{pk}\right) - 1.
\end{aligned}$$

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