# Preface

The project of writing this book was conceived and conceptualized in December 2008 during the *Centenary Conference* of the *Indian Institute of Science*, my Alma mater that has shaped my career, and life as well, for the past three and half decades. On December 16, 2008, Professor Eric Maskin who had received the 2007 Sveriges Riksbank prize (aka Nobel Prize in Economic Sciences) (jointly with Professors Leonid Hurwicz and Roger Myerson) gave a lively, lucid, enthralling, and inspirational talk entitled *Mechanism Design: How to Implement Social Goals* to an audience comprising more than 1500 scientists, engineers, and graduate students. Soon after this talk, it occurred to me that a book on game theory emphasizing not only non-cooperative games and cooperative games but also mechanism design would be valuable for engineering audience (in general) and computer science audience (in particular). I had been teaching a game theory and mechanism design course to our master's and doctoral students in computer science since 2004. This coupled with the brief but breathtakingly stimulating interaction with Professor Maskin sowed the seeds for undertaking this ambitious project of writing the book. It is therefore befitting that the book is dedicated to Professor Eric Maskin and his co-laureates Professors Leonid Hurwicz and Roger Myerson. This triumvirate, through their path-breaking work on mechanism design, have opened up this discipline to numerous powerful applications cutting across boundaries of disciplines.

Studying the rational behavior of entities interacting with each other in the context of a variety of contemporary applications such as Internet advertising, electronic marketplaces, social network monetization, crowdsourcing, and even carbon footprint optimization, has been the bread and butter of our research group here at the Game Theory Lab at the Department of Computer Science and Automation, Indian Institute of Science. Specifically, the application of game theoretic modeling and mechanism design principles to the area of Internet and network economics has been an area of special interest to the group for a decade now.

More than eight decades ago, the legendary John von Neumann played a significant role in the creation of two different exciting disciplines: *Game Theory* and *Computer Science*. Astonishingly, in the past fifteen years (1998-2013), There has been a spectacular convergence of the above two intellectual currents. The applications of game theory and mechanism design to problem solving in engineering and computer science applications have exploded in these fifteen years. This phenomenon certainly spurred us to dive into this area in the last decade.

Further, during this period, there were other developments that made sure we got locked into this area. Intel India, Bangalore, funded a collaborative project in 2000 that required the development of a multi-attribute combinatorial procurement auction for their indirect materials procurement. General Motors R & D, Warren, Michigan, next collaborated with our group to develop procurement auction mechanisms during 2002-2007. Meanwhile, Infosys Technologies, Bangalore, collaborated with us in 2006-07 on applying game theory and mechanism design to an interesting web services composition problem. The current collaboration with the Infosys team is focused on using game theory and mechanism design techniques to carbon footprint optimization. IBM India and IBM India Research Labs provided us with funding and a faculty award to make further explorations into this area. All this work culminated in a 2009 research monograph entitled *Game Theoretic Problems in Network Economics and Mechanism Design Solutions* (co-authored with my graduate students Dinesh Garg, Ramasuri Narayanam, and Hastagiri Prakash) and a string of research papers. We are also currently engaged with Xerox Research on fusing mechanism design with machine learning to extract superior performance from service markets. These projects have helped us to investigate deep practical problems, providing a perfect complement to our theoretical work in the area.

We have also been fortunate to be working in this area during an eventful period when game theorists and mechanism designers have been awarded the Nobel Prize in Economic Sciences. We were excited when Professors Robert Aumann and Thomas Schelling were awarded the Prize 2005. In fact, we had an illuminating visit by Robert Aumann in January 2007 to the Indian Institute of Science. We were delighted when, just two years later, Professors Leonid Hurwicz, Eric Maskin, and Roger Myerson were awarded the Prize in 2007 for their fundamental contributions to mechanism design theory. Finally, our excitement knew no bounds in October 2012 when Professors Lloyd Shapley and Professor Al Roth were announced as the winners of the prize for 2012.

### *Objectives of the Book*

Set in the above backdrop, this book strives to distill the key results in game theory and mechanism design and present them in a way that can be appreciated by students at senior undergraduate engineering level and above. The book includes a number of illustrative examples, carefully chosen from different domains including computer science, networks, engineering, and microeconomics; however they are fairly generic.

There are numerous excellent textbooks and monographs available on game theory. This book has drawn inspiration from the following reference texts: Mas-Colell, Whinston, and Green [1]; Myerson [2]; Nisan, Roughgarden, Tardos, and Vazirani [3]; Shoham and Leyton Brown [4]; Straffin [5]; Osborne [6]; and the very recent book by Maschler, Solan, and Zamir [7]. The dominating theme in many of the above texts is social sciences, particularly microeconomics. Our book is different in two ways. First, it has the primary objective of presenting the essentials of game theory and mechanism design to an engineering audience. Since I happen to be from a computer science department, there is also an inevitable emphasis on computer science based applications. Second, the book has a detailed coverage of mechanism design unlike most books on game theory. A precursor to this current book is an earlier monograph by Narahari, Garg, Narayanam, and Prakash [8].

## *Outline and Organization of the Book*

The book is structured into three parts: *Non-cooperative game theory* (Chapters 2 to 13); *Mechanism design* (Chapters 14 to 24); and *Cooperative game theory* (Chapters 25 to 31). Chapter 1 is an introduction to the book and Chapter 32 is an epilogue while Chapter 33 attempts to provide a succinct discussion of mathematical preliminaries required for understanding the contents of the book.

Each chapter commences with a motivation and central purpose of the chapter, and concludes with a crisp summary of key concepts and results in the chapter and a set of references to probe further. At the end of each chapter, a set of exercise problems is also included. In relevant chapters, programming assignments are also suggested. The book has a table of acronyms and notations at the beginning of the book. The book further contains, at relevant places, informative biographical sketches of legendary researchers in game theory and mechanism design. We now present a chapter-by-chapter outline of the book.

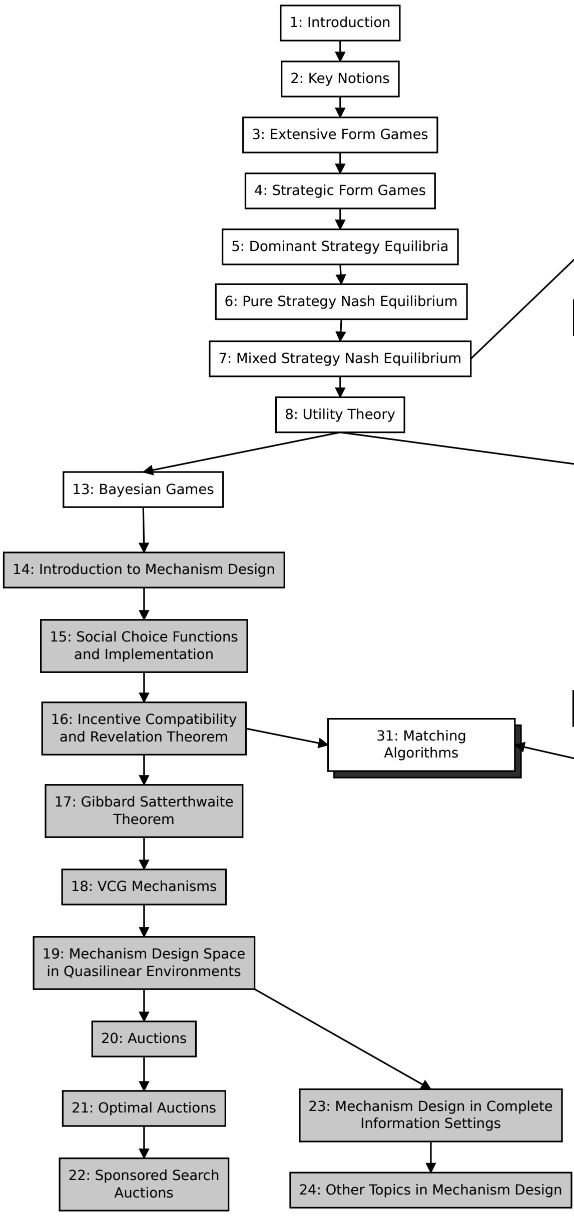
### *Chapter Reading Sequence*

The picture appearing overleaf depicts the sequential dependency among the main chapters of the book. The rectangles corresponding to Part 1, Part 2, and Part 3 are shaded differently in the picture. The diagram is self-explanatory. Since Chapter 32 (Epilogue) and Chapter 33 (Mathematical Preliminaries) have a special purpose, they are not depicted in the diagram.

### *Part 1: Non-cooperative Game Theory*

We first introduce, in Chapter 2, key notions in game theory such as *preferences*, *utilities*, *rationality*, *intelligence*, and *common knowledge*. We then study two representations for non-cooperative games: *extensive form representation* (Chapter 3) and *strategic form representation* (Chapter 4).

In Chapters 5,6, and 7, we describe different solution concepts which are fundamental to the analysis of strategic form games: dominant strategies and *dominant*




Reading sequence of chapters in the book

*strategy equilibria* (Chapter 5); *pure strategy Nash equilibrium* (Chapter 6); and *mixed strategy Nash equilibrium* (Chapter 7). In Chapter 8, we introduce the *utility theory* of von Neumann and Morgenstern which forms the foundation for game theory.

Chapters 9, 10, 11, and 12 are devoted to studies on existence and computation of Nash equilibria. In Chapter 9, we focus on two player zero-sum games. In

Chapter 10, we provide a detailed treatment of the Nash theorem that establishes the existence of a mixed strategy Nash equilibrium in finite strategic form games. Chapter 11 is concerned with algorithmic computation of Nash equilibria while Chapter 12 deals with computational complexity of finding Nash equilibria.

In Chapter 13, we introduce *Bayesian games* which are games with *incomplete information*. These games play a central role in mechanism design which is the subject of Part 2 of the book.

*Part 2: Mechanism Design*

Mechanism design is the art of designing games so that they exhibit desirable equilibrium behavior. In this part (Chapters 14-24), we study fundamental principles and key issues in mechanism design.

In Chapter 14, we introduce mechanisms with simple, illustrative examples and discuss the key notions of *social choice functions*, *direct mechanisms*, and *indirect mechanisms*. In Chapter 15, we bring out the principles underlying implementation of social choice functions by mechanisms. In Chapter 16, we define the important notion of incentive compatibility and bring out the difference between dominant strategy incentive compatibility (DSIC) and Bayesian incentive compatibility (BIC). We prove the *revelation theorem*, an important fundamental result. Chapter 17 is devoted to two key impossibility results: the Gibbard-Satterwaite theorem and the Arrow theorem.

Chapters 18-22 are devoted to different classes of quasilinear mechanisms which are either DSIC or BIC. In Chapter 18, we study VCG (Vickrey-Clarke-Groves) mechanisms, by far the most extensively investigated class of mechanisms. Chapter 19 is devoted to an exploration of mechanism design space in quasilinear environment, including Bayesian mechanisms. In Chapter 20, we discuss auctions which are a popular example of mechanisms. In Chapter 21, we study optimal mechanisms, in particular the Myerson auction. In Chapter 22, we study the sponsored search auction problem in detail to illustrate a compelling application of mechanism design.

In Chapter 23, we discuss *implementation in Nash equilibrium* which assumes a complete information setting. Finally, Chapter 24 provides a brief description of important advanced topics in mechanism design.

*Part 3: Cooperative Game Theory*

We commence our study of cooperative game theory in Chapter 25 with a discussion on *correlated strategies* and *correlated equilibrium*. The *Nash bargaining problem* represents one of the earliest and most influential results in cooperative game theory. Chapter 26 describes the problem and proves the Nash bargaining result. We introduce in Chapter 27, *multiplayer coalitional games* or *characteristic form games*. In particular, we introduce *transferable utility games* (TU games) with several illustrative examples.

Chapters 28-30 are devoted to solution concepts in cooperative game theory. In Chapter 28, we study *the core*, a central notion in cooperative game theory. The *Shapley value* is a popular solution concept that provides a unique allocation to a set of players in a cooperative game. In Chapter 29, we present the Shapley axioms and prove the existence and uniqueness of the Shapley value. In Chapter 30, we briefly study five other important solution concepts in cooperative game theory: *Stable sets*, *Bargaining sets*, *Kernel*, *Nucleolus*, and *Gately point*. Chapter 31 is devoted to the interesting topic of matching algorithms.

We conclude the book in Chapter 32 with some thoughts on how best to utilize the insights from the book. We have included, in Chapter 33, an appendix that contains key notions and results from probability theory, linear algebra, linear programming, mathematical analysis, and computational complexity, which are used in a crucial way at various points in this textbook.

### *Intended Audience*

The primary audience for the book include: senior undergraduate, first year master's, and first year research students studying computer science, networks, communications, electrical engineering, industrial engineering and operations research, microeconomics, and management science. Researchers and industry professionals who wish to explore game theory and mechanism design in Internet and network economics applications will find the book useful. After a thorough reading of this book, we expect that readers would be able to apply game theory and mechanism design in a principled and mature way to solve relevant problems. It is our sincere hope that the book will whet the appetite of the intended audience and arouse curiosity in this exciting subject. To provide an idea of how different types of audience could potentially benefit from this book, here are several examples:

- Computer science students will be able to make forays into topical areas such as algorithmic game theory, algorithmic mechanism design, computational social choice, auctions and market design, electronic commerce, Internet monetization, social network research, and mechanism design for multiagent systems.
- Computer science, electronics, and electrical engineering students would be able to explore research areas like network protocol design, dynamic resource allocation in networked systems, design of multiagent smart grid networks, and network science.
- Industrial engineering or management science students would be in a position to undertake research in supply chain network design, logistics engineering, dynamic pricing in e-business, etc.
- Researchers on inter-disciplinary topics such as cyberphysical systems, intelligent transportation, service science, green supply chains, and human

computation systems (such as crowdsourcing networks) would be able to formulate and solve topical problems using the tools covered in this book.

### *Possible Course Offerings*

I have taught for several years a course on game theory to master's and doctoral students at the Indian Institute of Science, Bangalore. About 80 percent of the students have been from a computer science background with the rest of the students drawn from communications, electrical engineering, and management. In fact the book can be considered as a culmination of my lovely experience with this course spread over a number of years. The lecture notes of the course have survived the scrutiny of the talented students and in fact many of the students have contributed to this book by providing critical comments and suggestions. The course taught by me typically covers about 60 percent each of the contents in Part 1 (Non-cooperative game theory); Part 2 (Mechanism design); and Part 3 (Cooperative game theory).

With a judicious selection of topics, it is possible to design several courses based on this book. We provide three possibilities below.

#### *Undergraduate Level Course on Game Theory*

To an audience consisting of third year or fourth year undergraduate students, the following collection of topics would make an interesting course.

- *Non-cooperative game theory*: Chapter 1, Chapter 2, Chapter 3, Chapter 4, Chapter 5, Chapter 6, Chapter 7, Chapter 9.
- *Cooperative game theory*: Parts of Chapter 25, Chapter 26, Chapter 27, Chapter 28, Chapter 29, Chapter 31.
- *Mechanism design* (optional): Parts of Chapter 13, Chapter 14, Chapter 15, Chapter 16, Chapter 20

#### *Master's Level Course on Game Theory*

To an audience consisting of final year undergraduate students, master's students, and first year graduate students, the entire book would be relevant. To cover the entire book as a one semester course would be challenging, so a judicious choice of topics will be the key.

#### *Graduate Level Course on Game Theory*

About 40 percent material of a graduate level course could be covered from this book. If the students have already gone through an undergraduate level course on game theory (as explained above), then the remaining chapters of this book (especially Chapter 10, Chapter 11, Chapter 12, Chapter 13, all chapters in mechanism design (Chapters 14-24), and all chapters in cooperative game theory (Chapters 25-31) would provide the initial content. Appropriate material from advanced books and from the current literature should complement and complete such a course offering.

### *Convention in Usage of Certain Common Words and Phrases*

We wish to draw the attention of the readers regarding use of certain words and phrases. We use the words *players* and *agents* interchangeably throughout the text. The words *bidders*, *buyers*, and *sellers* are often used to refer to players in an auction or a market. The words *he* and *his* are used symbolically to refer to both the genders. This is not to be mistaken as gender bias. Occasionally we have also used the words *she* and *her*. We have also sporadically used the words *it* and *its* while referring to players or agents.

### *Supplementary Resources*

The URL http://lcm.csa.iisc.ernet.in/hari/book.html will take the interested readers to supplementary material which would be continuously updated. The material includes additional references, additional problems, solutions to selected exercises, and viewgraphs for selected chapters.

### *Feedback is Welcome!*

No book is flawless. We invite you to report any flaws and provide your valuable comments and suggestions by sending email to me at hari@csa.iisc.ernet.in. We would be delighted to post the clarifications on the website at the URL mentioned above.

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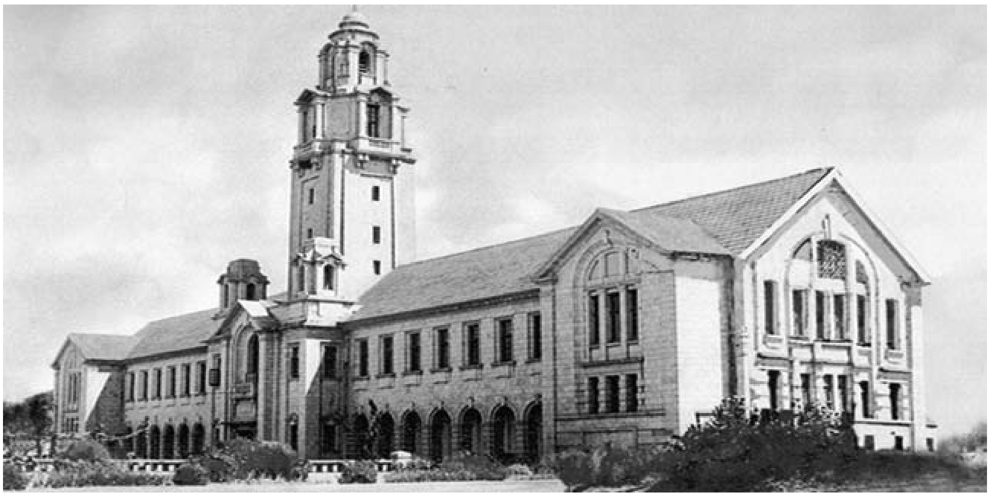

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