ESc 101: ALGORITHMS AND PROGRAMMING

Y. Narahari and M. Narasimha Murty
Indian Institute of Science
Bangalore, India
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Problem Set 4

1. Set up a recurrence relation to capture the number of comparisons made by bubble sort and show that the total number is $O(n^2)$ where n is the number of elements sorted, that is show that $T(n) = O(n^2)$.

2. Consider the pairs of integers i and j such that $1 \leq i < j \leq n$. Show that the total number of valid pairs is characterized by the following recurrence:

$$T(n) = (n-1) + T(n-1)$$
$$T(2) = 1$$

   Solve this recurrence to get the value of $T(n)$ in terms of n.

3. Consider binary search on a sorted collection of $n = 2^k$ elements stored in an array. Show that the recurrence is:

$$T(n) = T(n/2) + 1$$
$$T(1) = 1$$

   Solve the recurrence to compute $T(n)$ as a function of n.

4. Consider the analysis of average complexity of quicksort. Instead of choosing the pivot as the bigger of the first two elements, if we select the largest of the first three elements as pivot, then obtain the probability that the pivot is the $(i+1)^{st}$ element.

5. Give a simple example of a four node directed graph with some negative weight edges for which Dijkstra's algorithm produces incorrect answers.

6. Is there a directed graph with some negative weight edges which produces correct answer?

7. Consider an Undirected graph $G = (V, E)$ with n vertices. Show how a one dimensional array can be used to represent G. What is the size of the array?

8. Let (u, v) be a minimum-weight edge in an undirected graph G. Show that (u, v) belongs to some minimum spanning tree of G.

9. Let e be maximum-weight edge on some cycle of an undirected graph $G = (V, E)$. Prove that there is a minimum spanning tree of $G' = (V, E - \{e\})$ that is also the minimum spanning tree of G.

10. For an undirected graph G with n vertices and e edges, show that $\sum_{i=1}^{n} d_i = 2e$ where d_i is the degree of vertex i (number of edges incident on i).