Electronic Commerce

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Problem Set 3

1. Give examples of two player games for the following situations

   (a) The game has a unique Nash equilibrium which is not a weakly dominant strategy equilibrium

   (b) The game has a unique Nash equilibrium which is a weakly dominant strategy equilibrium but not a strongly dominant strategy equilibrium

   (c) The game has one strongly dominant, one weakly (but not strongly) dominant strategy equilibrium and a third one which is only a Nash equilibrium

2. The following payoff matrix corresponds to a modified version of the Prisoner's Dilemma problem called the DA's brother problem. In this problem prisoner 1 is related to the District Attorney. How is this problem different? Does it have a strongly dominant or a weakly dominant equilibrium?

	2	
1	NC	C
NC	0, −2	−10, −1
C	−1, −10	−5, −5

3. There are n departments in I.I.Sc. Each department can try to convince the Director to get a certain budget. If h_i is the number of hours of work put in by a department to make the proposal and $c_i = w_i h_i^2$ is cost of this effort to the department, where w_i is a constant. When the effort levels of the departments are $(h_1, h_2, \ldots, h_n)$, the total budget that gets allocated to all the departments is:

$$\alpha \sum_{i=1}^{n} h_i + \beta \prod_{i=1}^{n} h_i$$

   where α and β are constants.

   Consider a game where the departments simultaneously and independently decide how many hours to spend on this effort. Show that a strictly dominant strategy equilibrium exists iff $\beta = 0$. Compute this equilibrium.

4. Argue that if a player has two weakly dominant strategies, then for every strategy profile of the other players, the two strategies yield him the same payoff.

5. Consider the following instance of the prisoners' dilemma problem.

	C	NC
C	(-4, -4)	(-2, -x)
NC	(-x, -2)	(-x, -x)

Find the values of x for which:

(a) the profile (C,C) is a strongly dominant strategy equilibrium.

(b) the profile (C,C) is a weakly dominant strategy equilibrium but not a strongly dominant strategy equilibrium.

(c) the profile (C,C) is a Nash equilibrium but not a dominant strategy equilibrium.

(d) the profile (C,C) is not even a Nash equilibrium.

In each case, say whether it is possible to find such an x. Justify your answer in each case.

6. Compute a Nash equilibrium for the two person game with

$$S_1 = \{0, 1\} \qquad S_2 = \{3, 4\}$$

$$u_1(x, y) = -u_2(x, y) = |x - y| \quad \forall (x, y) \in [0, 1] \times [3, 4]$$

[Jones 94]

7. Consider the game $(N, (S_i), (u_i))$ when $N = \{1, \ldots, n\}$ and $S_i = \{1, \ldots, 1n\} \quad \forall i \in N$.

$$\begin{aligned} u_1(s_i, \ldots, s_n) &= a_{ik} > 0 \qquad \text{if } s_1 = \cdots = s_n = k \\ &= 0 \qquad\qquad\quad \text{otherwise} \end{aligned}$$

show that the only pure strategy profiles which are not equilibrium points are those with exactly $(n - 1)$ of the s_i equal.

8. Find the pure strategy Nash equilibrium of the following game.

	X	Y	Z
A	6, 6	8, 20	0, 8
B	10, 0	5, 5	2, 8
C	8, 0	20, 0	4, 4

9. Find the pure strategy Nash equilibria for the following two player game.

	A	B	C	D
A	5, 2	2, 6	1, 4	0, 4
B	0, 0	3, 2	2, 1	1, 1
C	7,0	2, 2	1, 5	5, 1
D	9, 5	1, 3	0, 2	4, 8

10. Consider a procurement auction for a single indivisible good with a buyer (agent 0) and two suppliers (agent 1 and agent 2). Each supplier submits a sealed bid and whoever submits the smaller bid will win the auction. If both of them submit the same bid, then agent 2 is announced as the winner. The winning agent will receive a sum equal to the bid. Assume that the bids from the agents are independent draws from the uniform distribution on [0,1]. For this scenario, under a quasi-linear setting, write down the social choice function being implemented. What is the underlying mechanism. Write down the components of the Bayesian game induced by this mechanism.

11. For the second price sealed bid auction with n buying agents, write down the Bayesian game induced by the mechanism. How will this game change if we have a procurement auction with one buyer and n sellers.

12. For the first price sealed bid auction with n buying agents, write down the Bayesian game induced by the mechanism. How will this game change if we have a procurement auction with one buyer and n sellers.

13. In the case of Revenue equivalence theorem, under the benchmark model, write down the Bayesian games underlying the English Auction, Dutch auction, First price Auction and Vickrey Auction.

14. Show that if $f : \Theta \to X$ is truthfully implementable in dominant strategies when the set of possible types is Θ_i for $i = 1, \ldots, I$, then when each agent i's set of possible types is $\hat{\Theta}_i \subset \Theta_i$ (for $i = 1, \ldots, I$,) the social choice function $\hat{f} : \hat{\Theta}_i \to X$ satisfying $\hat{f}(\theta) = f(\theta)$ for all $\theta \in \hat{\Theta}_i$ is truthfully implementable in dominant strategies.

15. A buyer and a seller are bargaining over the sale of an indivisible good. The buyer's valuation is $\theta_b = 10$. The seller's valuation takes one of two values: $\theta_s \in \{0, 9\}$. Let t be the period in which trade occurs ($t = 1, 2, \ldots$) and let p be the price agreed. Both the buyer and the seller have discount factor $\delta < 1$.

    (a) What is the set X of alternatives in this setting?

    (b) Suppose that in a Bayesian Nash equilibrium of this bargaining process, trade occurs immediately when the seller's valuation is 0 and the price agreed to when the seller has valuation θ_s is $(10 + \theta_s)/2$. What is the earliest possible time at which trade can occur when the seller's valuation is 9 ?

16. Write down the allocation rule and payment rule in Clarke's mechanisms. Using these, derive the allocation rule and payment rule of Vickrey auction (that is, second price sealed bid auction).